Qualifying Exam, Spring 2021
Solid Mechanics

* This is a closed-book test (with cheat sheets included), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #_____, #_____, and #_____ to be graded.

Problem 1. An isotropic elastic sphere (Young's modulus 20& GPa, shear modulus 79.2 GPa) of radius 5 cm is under the uniform stress field

$$\boldsymbol{\sigma} = \begin{bmatrix} 6 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$$

Find the change of volume for the sphere.

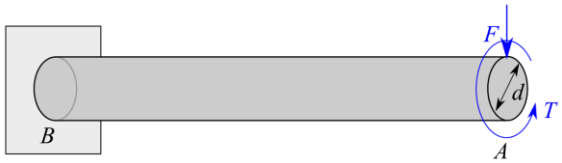
Problem 2. For any stress state $\boldsymbol{\sigma}$ we define the deviatoric stress $\boldsymbol{s}$ to be $s_{ij} = \sigma_{ij} - (\sigma_{kk}/3)\delta_{ij}$, where σ_{kk} is the first invariant of the stress tensor $\boldsymbol{\sigma}$.

(a) Calculate the first invariant of the deviatoric stress.

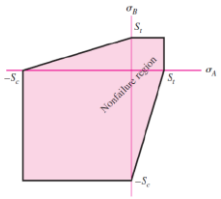
(b) Evaluate deviatoric stress for

$$[\sigma] = \begin{bmatrix} 100 & 500 & -200 \\ 500 & 300 & 400 \\ -200 & 400 & 900 \end{bmatrix} kPa$$

(c) Show that the principal directions of the stress tensor coincide with those of the deviatoric stress tensor.



Quadrant condition	Failure criterion
$\sigma_A \geq \sigma_B \geq 0$	$\sigma_A = \frac{S_{ut}}{n}$
$\sigma_A \geq 0 \geq \sigma_B$	$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$
$0 \geq \sigma_A \geq \sigma_B$	$\sigma_B = -\frac{S_{uc}}{n}$

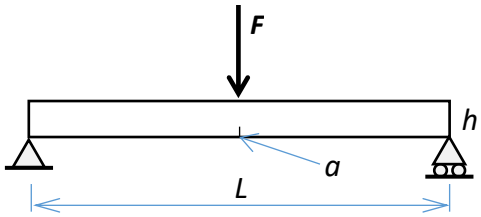


Problem 3. Given a cantilever beam of circular cross section subjected to a shear force F and torque $T = 2F \cdot l_{AB}$, calculate the maximum value of F.

Consider the material to be brittle with ultimate tensile and compressive strengths $S_{ut} = 40\ kpsi$ and $S_{uc} = 60\ kpsi$. Use Coulomb-Mohr failure criterion, see failure envelope on the left.

Use length $l_{AB} = 20\ in$, diameter $d = 1.2\ in$, and safety factor $n = 2$.

Neglect shear stress due to shear force. Make sure to indicate the point in the beam with max effective stress



Problem 4. A simply-supported beam of solid square cross section with a side length $h = 3\ cm$ and a total length $L = 1.5\ m$ is subjected to a static point force $F = 1000\ N$ in the middle.

Calculate:
1. the safety factor against fracture. Assume initial crack length $a = 3\ mm$.

2. the critical crack length for the given force. Assume the geometry factor β does not deviate significantly from the value given below for the considered range of crack lengths.

Use fracture toughness of the material $K_{IC} = 16\ MPa\sqrt{m}$, and stress intensity modification factor (aka geometry factor) $\beta = 1.1$.

Focus on mode I only. Make sure to show the bending moment diagram.