Space Dynamics and Controls Problems for Qualifying Exam - Fall 2013

(solve only 3 from 4 problems!!)

Problem 1

A satellite in Earth orbit passes through its perigee point at an altitude of 200 km above the Earth's surface and at a velocity of 7,850 m/s. Calculate the apogee altitude of the satellite and the eccentricity of the orbit. Consider the Earth to be perfectly spherical with a radius of $R_E = 6378$ km and the gravity parameter for the Earth to be $\mu_E = 398600$ km^3/s^2.

Problem 2

A satellite is in an Earth orbit of eccentricity $e = 0.00132$, inclination $i = 89.1°$, and argument of periapsis $\omega = 261°$ in a geocentric equatorial frame. Its perigee altitude is $z_p =917$ km. What is the maximum altitude achieved by the satellite above the equator at any time in its orbit? Let the right ascension of the ascending node of the orbit be $\Omega = 27°$. Find the position vector of the satellite at its highest point above the equator in the geocentric equatorial frame [Assume the Earth to be spherical and uniform, with its radius equal to 6378 km].

Problem 3

Consider a rigid dumbbell-shaped satellite in motion around a spherical Earth, with gravitational parameter $\mu_E = GM_E$, where G is the universal gravitational constant and M_E denotes the mass of the Earth. The satellite is idealized as two point mass particles of mass m each at the two ends of a rigid massless link of length $2l$. Let b denote the position vector of the center of the satellite in an Earth-centered inertial coordinate frame and let its attitude be given by R, which is the rotation matrix from the Earth-centered inertial frame to a right-handed coordinate frame fixed at the center of the dumbbell satellite with its x-axis along the length of the rigid link connecting the end masses. Consider $m \ll M_E$.

(a) Give mathematical expressions for the Newtonian gravity force vector and gravity gradient moment vector due to the Earth on the dumbbell satellite. Express these vectors in both the Earth-centered inertial frame and the coordinate frame fixed at the center of the dumbbell satellite.

(b) Assume that there are no control or other external forces or moments on the satellite besides those due to gravity. Make use of your results in part (a) to obtain differential equations of motion for the position vector and the attitude (orientation) of the satellite with respect to the Earth-centered inertial frame. The differential equations should be expressed in terms of these quantities and their first and second time derivatives; include physical constants as necessary.

Problem 4

Consider a rigid body that is constrained to rotate about a fixed axis given by the unit vector $\hat{e} = [0.7032 \quad -0.1051 \quad -0.7032]^{\mathrm{T}}$ with a constant angular velocity vector $\omega = 0.05\hat{e}$ rad/s. The orientation of the rigid body with respect to an inertial frame is given by the direction cosine matrix $C(t)$ at time t, which transforms vectors in the inertial frame to vectors in the body coordinate frame. Assume that the body coordinate frame is initially aligned with the inertial frame, i.e., the initial direction cosine matrix is the identity matrix ($C(0) = I$). Obtain an analytical expression for $C(t)$ using Rodrigues formula, given the constant angular velocity ω. Express the rotational kinematics using the set of 3-2-1 Euler angles describing the (time-varying) direction cosine matrix. Explain why the 3-2-1 Euler angle description becomes singular despite an existing analytic solution to $C(t)$ in this case.

Space Dynamics and Controls Problems for Qualifying Exam - Fall 2012

Problem 1

Consider a rigid spacecraft of mass m, and inertia matrix J expressed in a spacecraft body-fixed coordinate frame, in the proximity of a planet of mass M with spherical mass distribution. Assume that $M \gg m$, an inertial coordinate frame is fixed to the center of the planet, and the spacecraft and planet are isolated from other gravitational influences. Does the center of mass and center of gravity of the spacecraft coincide in general? Obtain an expression for the gravity gradient moment on the spacecraft, given the position vector of the center of mass of the satellite (define all other notation used to express these quantities). Using either Newtonian mechanics or variational mechanics, express the translational and rotational dynamics equations of the spacecraft in its body coordinate frame.

Problem 2

A satellite is in an Earth orbit of eccentricity $e = 0.00132$, inclination $i = 89.1°$, and argument of periapsis $\omega = 261°$ in a geocentric equatorial frame. Its perigee altitude is $z_p = 917$ km. What is the maximum altitude achieved by the spacecraft above the equator at any time in its orbit? [Assume the Earth to be spherical and uniform, with its radius equal to 6378 km].

1. Low Earth (LEO) orbits generally have a range in **altitude** between 150 km and 1000 km.   The Earth's equatorial radius is 6378 km, and the Earth's gravitational parameter is 398,600 km^3/s^2.

a) Find the corresponding range of **periods** (time for one orbit) for circular LEO orbits in minutes.

b) Find the corresponding range of **speeds** for circular LEO orbits.

c) Find the corresponding range of **specific energy** for circular LEO orbits.

d) For an **elliptic** LEO orbit with minimum and maximum altitudes equal to the bounds on the altitude range given above, find the **specific energy, period,** and **eccentricity** of the orbit as well as the **speeds at perigee and apogee**.

2. Euler's equations for rotational motion of a rigid spacecraft are given as

$$[I]\dot{\vec{\omega}} = -[\tilde{\omega}][I]\vec{\omega} + \vec{L}$$

where $[I]$ is the inertia matrix, $\vec{\omega} = (\omega_1 \ \omega_2 \ \omega_3)^T$ is the angular velocity vector in body-frame coordinates, and $\vec{L}$ is the applied torque.

a) Write out the three scalar equations which are equivalent to the above equation *assuming that the body-fixed coordinate system is aligned with the principal body axes*.

b) For the torque-free $(\vec{L} = 0)$ case of the scalar equations obtained in a), now assume the body is axially symmetric $(I_1 = I_2 = I_T)$ and spinning about the $\hat{b}_3$ body axis with constant speed ω_3 where I_1, I_2, I_3 are the principal moments of inertia. Assuming a non-zero nutation angle, obtain the precession frequency (i.e. the rate at which the angular velocity vector sweeps around the angular momentum vector) in terms of I_3, I_T, ω_3.

c) Show that the resulting motion in b) is stable regardless if $I_3 < I_T$ or $I_3 > I_T$.