Vibrations Qualifying Exam
Study Material

The candidate is expected to have a thorough understanding of engineering vibrations topics. These topics are listed below for clarification. Not all instructors cover exactly the same material during a course, thus it is important for the candidate to closely examine the subject areas listed below. The textbook listed below is a good source for the review and study of a majority of the listed topics. One final note, the example problems made available to the candidates are from past exams and do not cover all subject material. These problems are not to be used as the only source of study material. The topics listed below should be your guide for what you are responsible for knowing.

Suggested textbook:

Theory of Vibrations with Applications, W. Thompson, (Prentice-Hall)

Topic areas:

1. Oscillatory Motion
2. Free Vibration
3. Harmonically Excited Vibration
4. Transient Vibration
5. Systems with Multiple Degrees of Freedom
6. Properties of Vibrating Systems
7. Lagrange's Equations
8. Vibration of Continuous Systems

Name ____________________________ **Spring 2008**

Qualifying Exam: Vibrations

CLOSED BOOK

This portion of the qualifying exam is **closed** book. You may have a calculator.

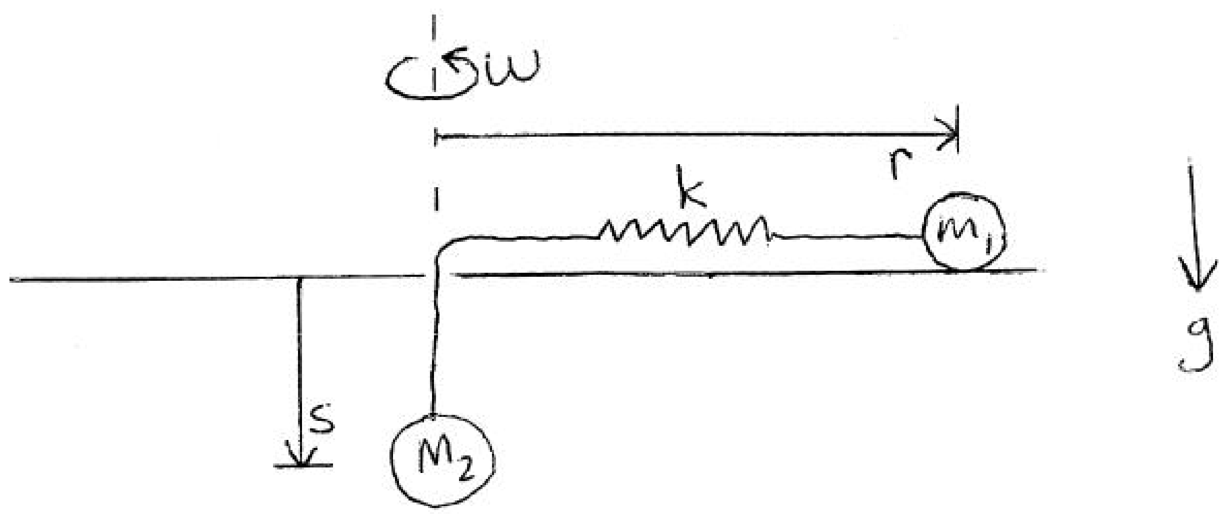
Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

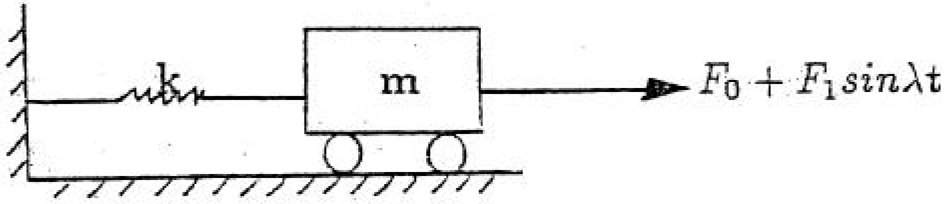
Be sure to put your name on all papers handed in, including this cover sheet.

1. The table on which m_i lies spins about a vertical axis at constant velocity ω as shown below. The coordinates of the masses are r and s as shown, and b is the length of the inextensible string plus the unstretched length of the spring.

   a. Use Lagrange's equations to derive the equations of motion.
   b. Obtain the linearized homogeneous equations in matrix/vector form and evaluate the stability of the system.



2. Determine the (complete) general solution for the system shown below. The force is applied at time $t = 0$.



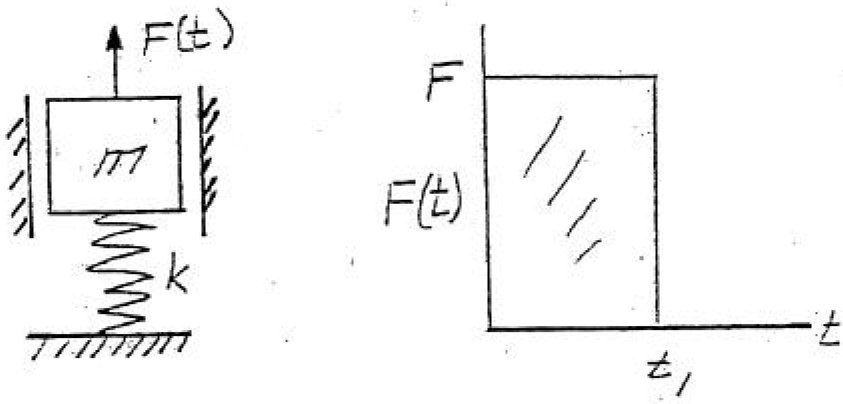
Name ____________________________ **Spring 2008**

Qualifying Exam: Vibrations

CLOSED BOOK

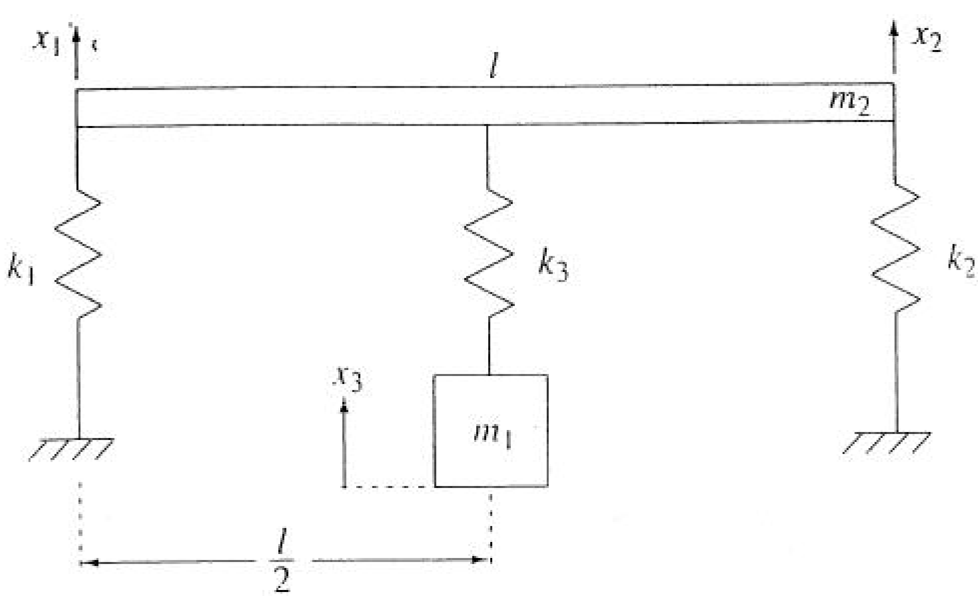
3. .A force $F(t)$ is suddenly applied to the mass-spring system shown below. The constant force is removed suddenly at $t = t_1$. Determine the response of the system when $t > t_1$.

$$x = (1/\omega_n)\int_0^t f(\tau)\sin[\omega_n(t-\tau)]d\tau$$



4. Determine the eigenvectors and natural frequencies for the system show below. The bar is uniform. Discuss the physical meaning of the responses.

   m_1 = 100 kg, m_2 = 1500 kg, k_1 = 10,000 N/m, k_2 = 12,000N/m, k_3 = 70,000 N/m, l = 4 m.



Name ______________________________ **Summer 2007**

Qualifying Exam: Vibrations

CLOSED BOOK

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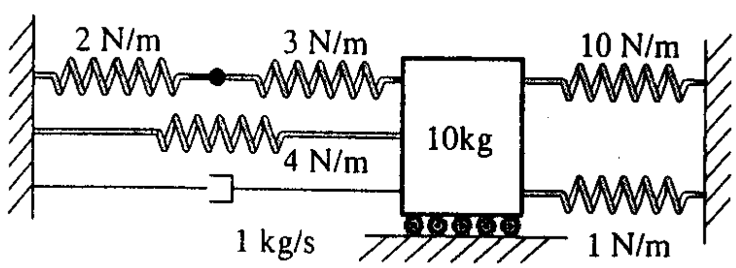
Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

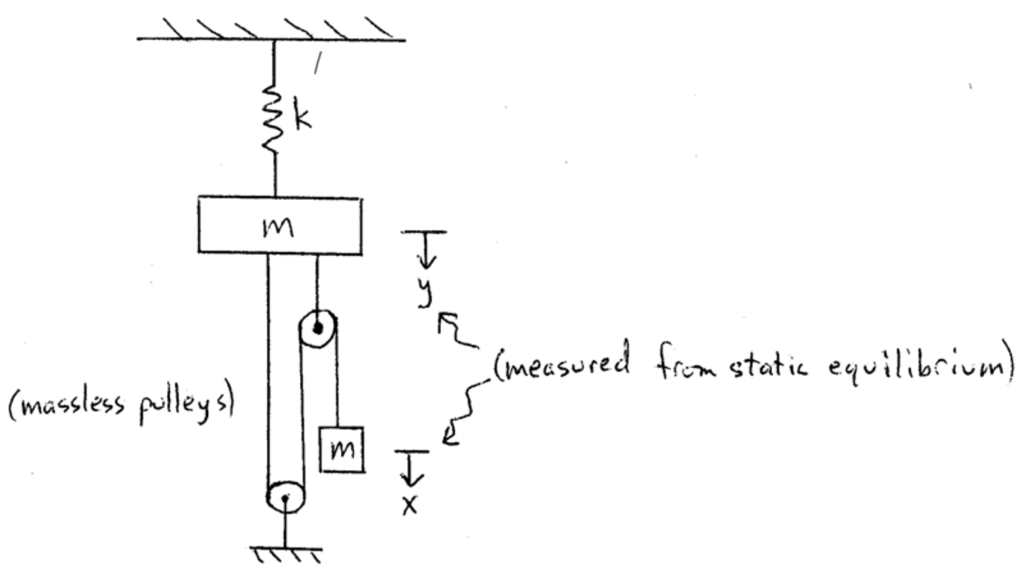
I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

1. Calculate the undamped natural frequency and damping ratio for the system below. Specify the damping type (under-, over-, or critically-damped). If under-damped, give the damped frequency of oscillation.



2. Use free body diagrams and Newton's laws to derive the equation of motion for the following pulley system. What is the natural frequency?

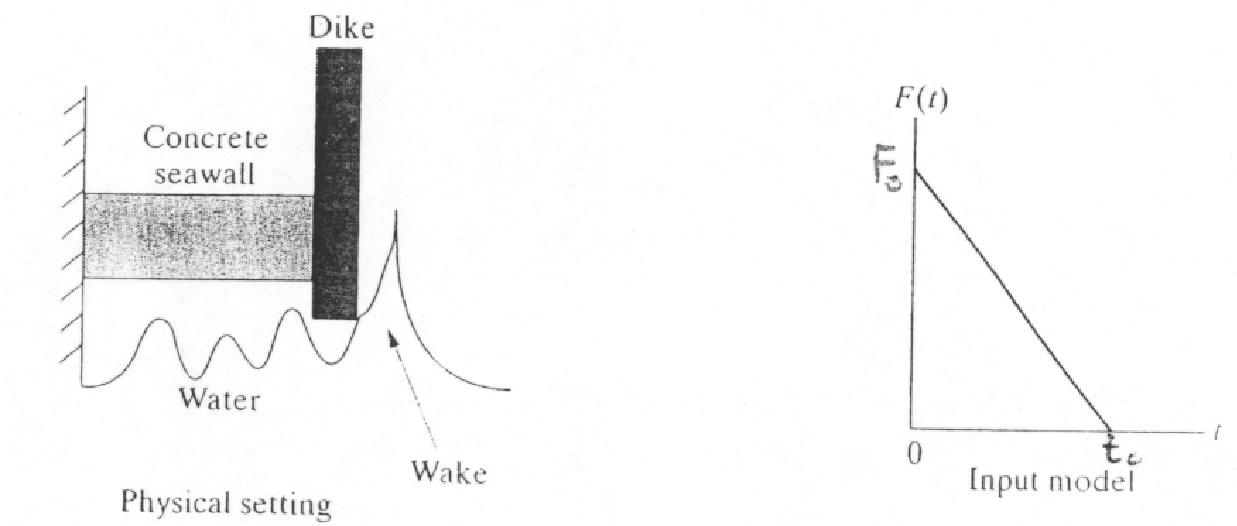


Name ____________________________ Summer 2007

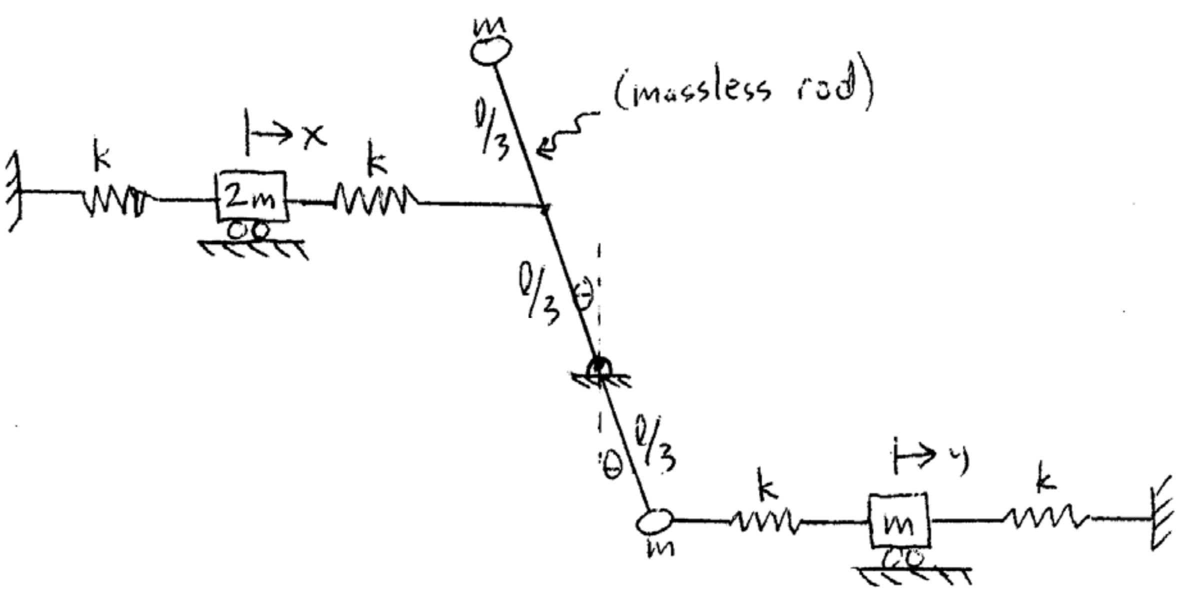
Qualifying Exam: Vibrations

CLOSED BOOK

3. A wave impacts a dike and seawall which has length l, area A, and Young's modulus E. Calculate the resulting vibration if the wave force is modeled as shown below and the seawall is modeled as an undamped single-degree-of-freedom system.



4. Use Lagrange's equations to derive the equations of motion for the following system. Obtain linearized equations about the stable equilibrium and write in matrix/vector form. Obtain the stability condition for the inverted pendulum.



NAME: ______________________ Spring 2006

Qualifying Examination
Subject: Vibrations

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Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

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Lagrange's Equations:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} + \frac{\partial V}{\partial q_j} = Q_j \qquad \delta U = \sum Q_j \delta q_j$$

Convolution Integral:

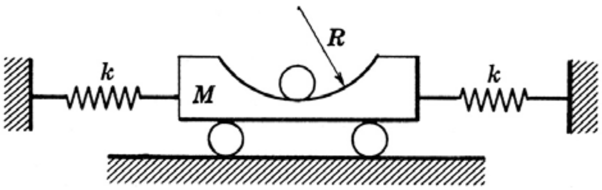
$$x(t) = \int F(\tau) h(t-\tau) d\tau$$

NAME: ______________________ Spring 2006

Qualifying Examination
Subject: Vibrations

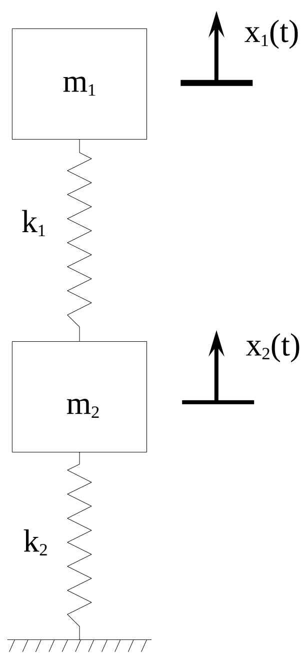
Problem 1:

The circular cylinder has a mass m and radius r, and rolls without slipping inside the circular grove of radius R, as shown below. Derive the equations of motion.



Problem 2:

A model of a vehicle suspension is given below. Write (a) the equations of motion in matrix form, (b) calculate the natural frequencies, and (c) determine the mode shapes for $k_1 = 10^3$ N/m, $k_2 = 10^4$ N/m, $m_2 = 50$ kg, and $m_1 = 2000$ kg.

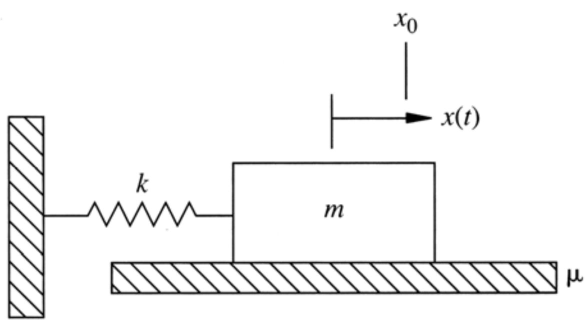


NAME: ______________________ Spring 2006

Qualifying Examination
Subject: Vibrations

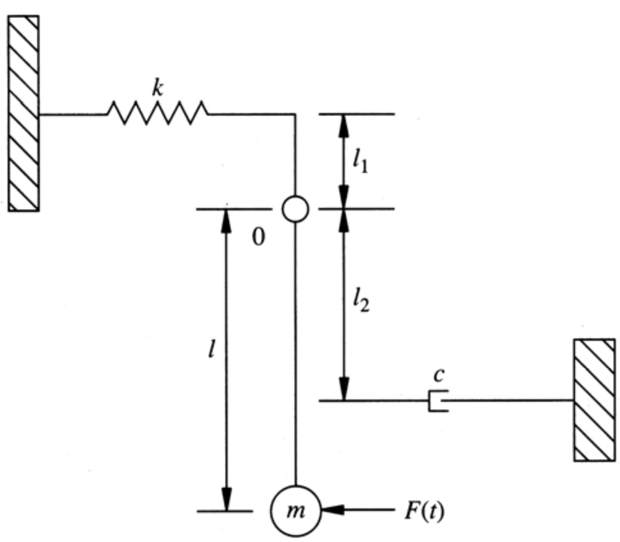
Problem 3:

The spring mass system is sliding on a surface with a kinetic coefficient of friction μ. For the initial conditions $x(0) = x_0$ and $\dot{x}(0) = 0$, determine the response of the system for one complete cycle of motion.



Problem 4:

Consider the pendulum mechanism below, which is pivoted at point O. k = 4x10^3 N/m, l_1 = 1.5 m, l_2 = 0.5 m, l = 1 m, and m = 40 kg. The mass of the beam is 40 kg; it is pivoted at point O and assumed to be rigid. Design the dashpot (i.e. calculate c) so that the damping ratio of the system is 0.2. Also determine the amplitude of vibration of the steady-state response if a 10-N force is applied to the mass, as indicated in the figure, at a frequency of 10 rad/s.



NAME: ______________________ Fall 2004

Qualifying Examination
Subject: Vibrations

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

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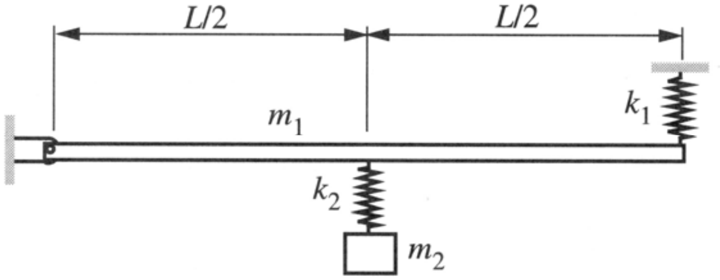
Problem 1:

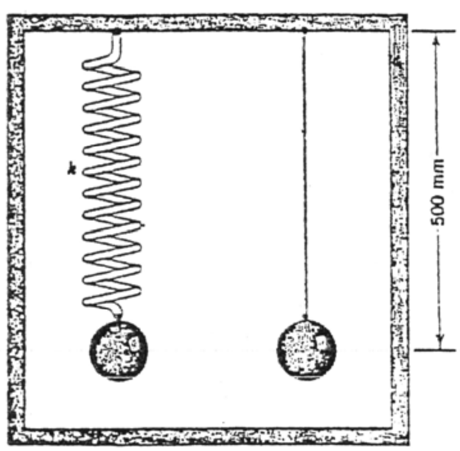
Determine the natural frequencies and corresponding mode shapes for the system shown below, where $k_1 = k_2 = k$ and $m_1 = m_2 = m$.



Problem 2:

Two 1-kg spheres are suspended from the roof of an elevator, as shown. When the elevator is at rest the period of extensional free vibration of the spring supported sphere is the same as that for the rotation of the pendulum. Determine:

(a) the stiffness k of the spring and
(b) the period of each system when the elevator accelerates upward at $a = 0.6g$. ($g = 9.807\ m/s^2$)

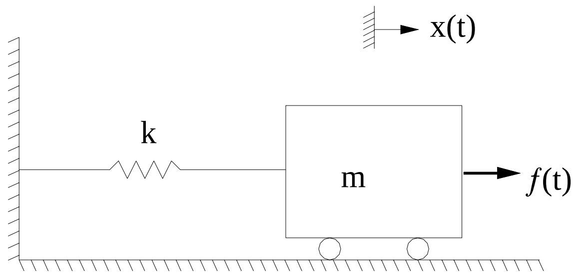


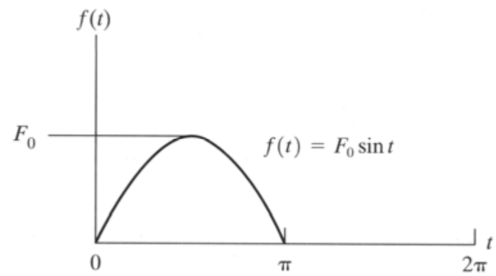
NAME: ______________________ Fall 2004

Problem 3:

The spring-mass mechanism shown in the figure below is subject to a force f(t) as shown. Determine the motion of the mass if the mass is at rest at the instant the force is applied.

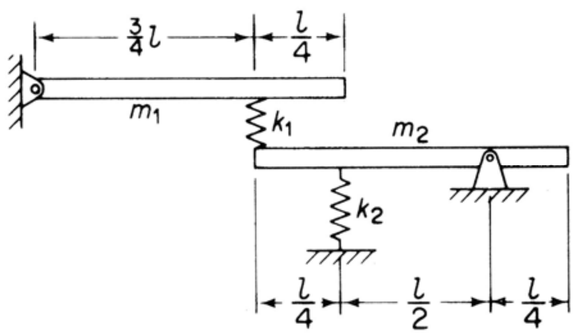
$$x_p = \left(\frac{1}{m\omega}\right)\int_0^t F(\tau)\sin\omega(t-\tau)d\tau$$





Problem 4:

Using Lagrange's equations, determine the equations of motion for the two uniform bars shown. Note that they are of equal length but different masses.



NAME: ______________________ Fall 2003

Qualifying Examination
Subject: Vibrations

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

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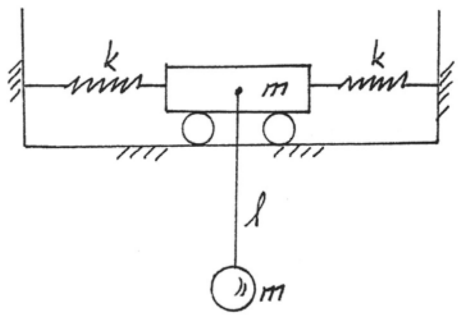
Problem 1:

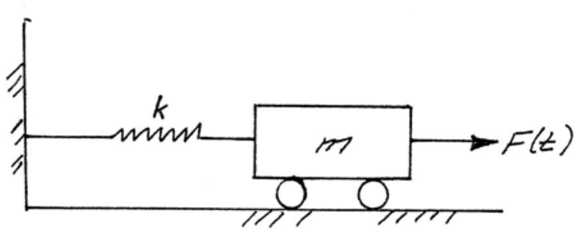
At t = 0 the block in the system is given a velocity $\mathbf{v_o}$ to the left. Determine the system's equations of motion and natural frequencies.

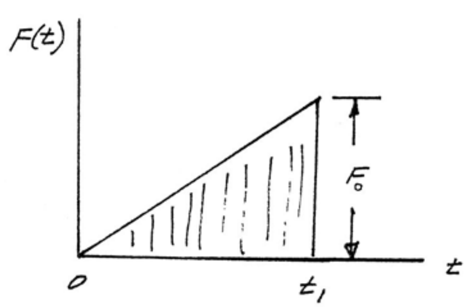


Problem 2:

The spring-mass mechanism shown in the figure is subject to a force F(t) as shown. Determine the expression for the motion of the mass for t > t1 if the mass is at rest at the instant the force is applied.

$$x_p = \left(\frac{1}{m\omega}\right)\int_0^t F(\tau)\sin\omega(t-\tau)\,d\tau$$

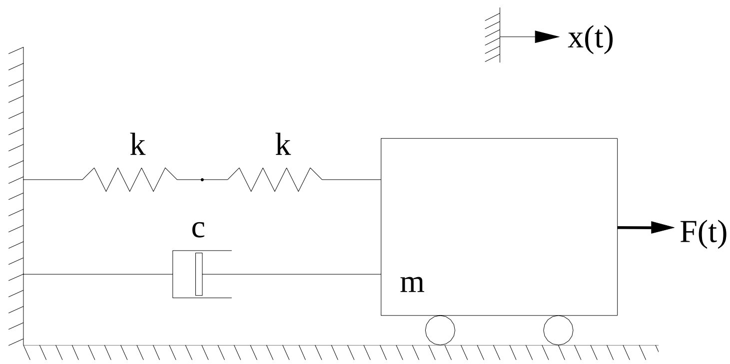




NAME: ______________________ Fall 2003

Problem 3:

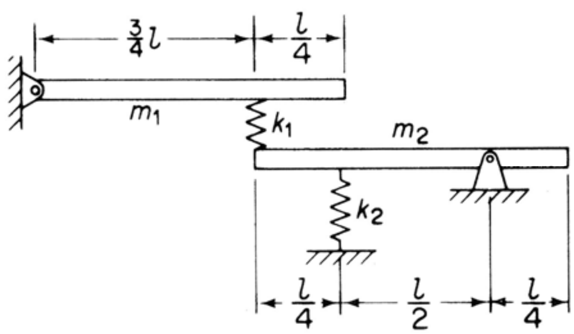
Consider the system below, m = 2kg, k = 100 N/m, and c = 40 N·s/m, F_o = 4N. Calculate the response, **x(t)** of the system to the applied load.



Initial Conditions
$x(0) = 0.1 \text{ m}$
$\dot{x}(0) = 0 \text{ m/s}$
Forcing Function
$F(t) = 10\cos(5t) + F_o$

Problem 4:

The two uniform bars shown below are of equal length but of different masses. Determine the equations of motion, natural frequencies, and mode shapes. Mass $m_2 = 2m_1$.



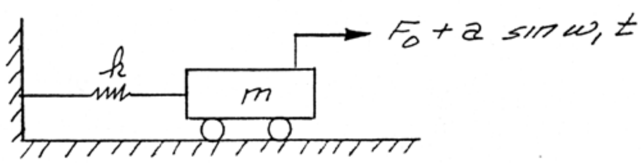
Qualifying Exam Spring 2003
Vibrations

This portion of the qualifying exam is **closed** book. You may have a calculator.

There are 4 problems here. Pick any 3 to work.

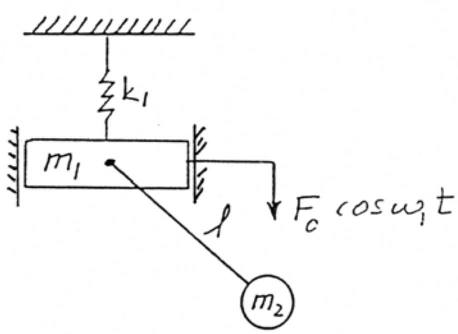
Problem 1:

Solve for the resulting motion if the force $F_o + a\sin(\omega_1 t)$ is suddenly applied to the system at time t =0. (Assume $\omega_1^2 \neq \frac{k}{m}$)



Problem 2:

Find the steady state solution for the forced system shown.

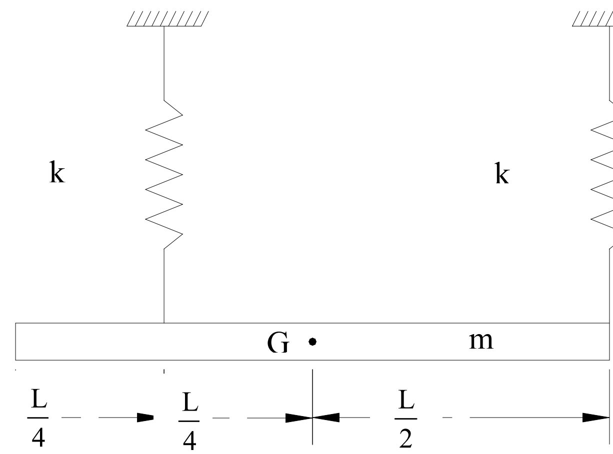


Problem 3:

For the system below determine (a) the equations of motion in matrix form, (b) the natural frequencies, (c) the mode shapes, and (d) write the equations that describe the displacement use constants for the unknown quantities with the appropriate subscripts.

$$I = \frac{1}{12} mL^2$$

Assume that the system below is shown in its equilibrium position. Use the vertical displacement of G as one of the degrees-of-freedom.

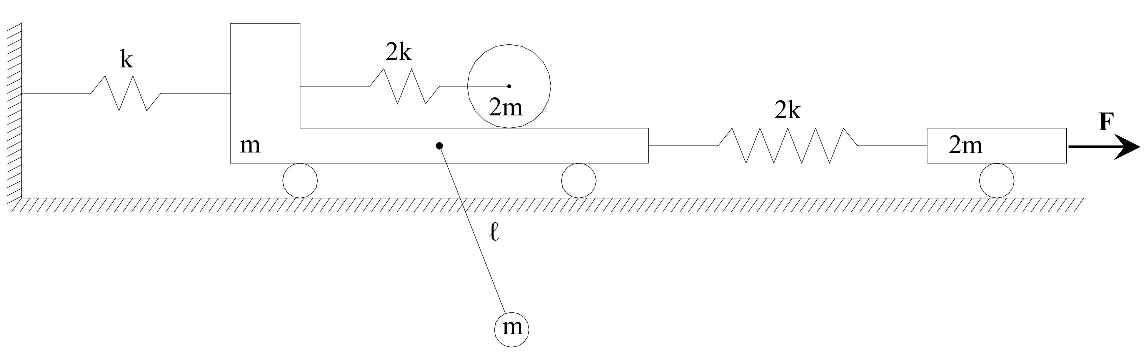


m = 4
k = 40
L = 8

Problem 4:

For the system below, determine the equation of motion in matrix form using Lagrange's equation.

$I = \frac{1}{2} mr^2$ for the disk.



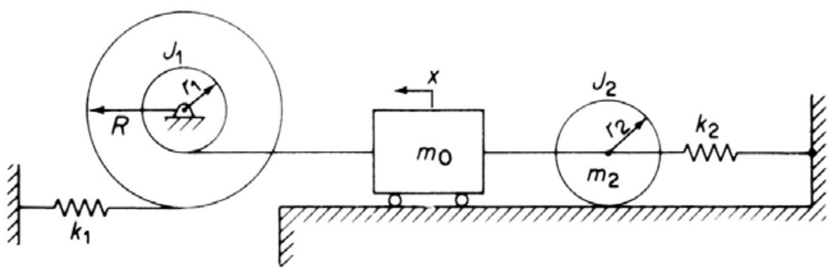
Qualifying Exam Fall 2002
Vibrations

This portion of the qualifying exam is **closed** book. You may have a calculator.

There are 4 problems here. Pick any 3 to work.

Problem 1:

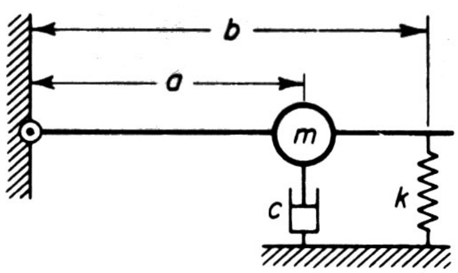
Determine the equation of motion of the system shown below in terms of the displacement *x*. Determine the expression for the system's natural frequency.



Problem 2:

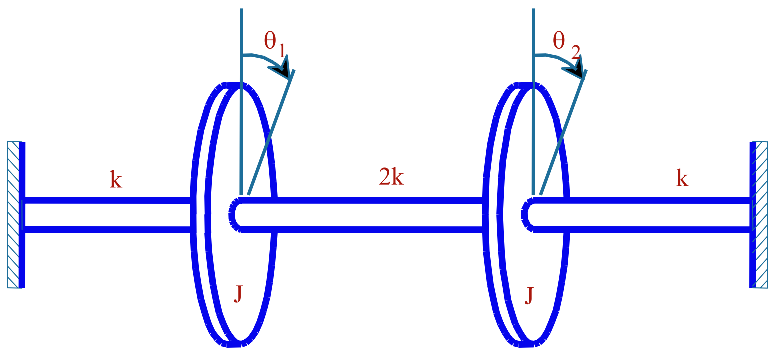
For the system shown determine:

a) the equation of motion
b) the natural frequency of damped oscillation
c) the critical damping coefficient.



Problem 3:

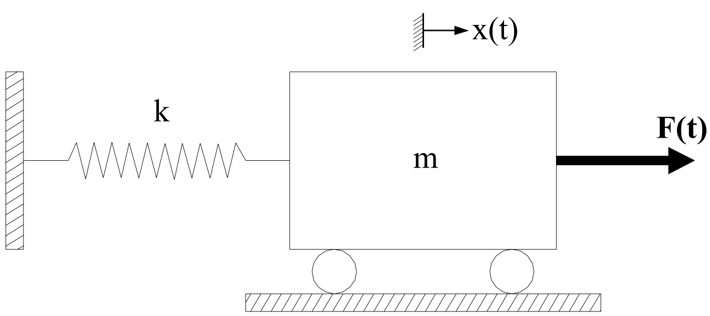
For the systems shown below, determine the natural frequencies and mode shapes. You may leave the solution in terms of J and k.

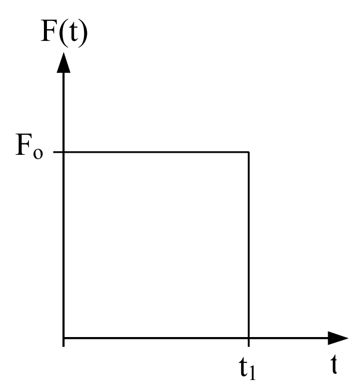


Problem 4:

The system show below is excited by the square pulse shown below. Determine the response of the system x(t) to the excitation pulse. The response of the system due to a unit impulse excitation at t = 0 is given by:

$$h(t) = \frac{1}{m\omega_n} \sin \omega_n t$$



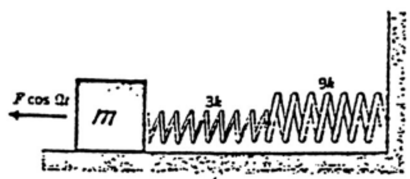


Qualifying Exam Fall 1998
Vibrations

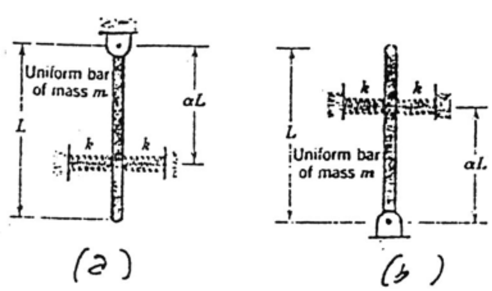
This portion of the qualifying exam is **closed** book. Note: During this time the exam was a vibrations and controls exam, so the controls problems are not included.

There are 4 problems here. Pick any 3 to work.

Problem #1. The harmonic force **F cos Ωt** is applied to the block at t=0. Initally, the block was at rest in the position where the compressive force in the series connected spring is **F**. Determine
(a) the response of the block, and
(b) the amplitude and phase angle of the steady state response for $\Omega = (k/m)^{1/2}$



Problem #2. Determine the natural frequency of each of the systems shown. Are there any values of the spring stiffness k for which either system will not oscillate freely about the vertical position?



$I_{G(rod)} = m\ell^2/12$

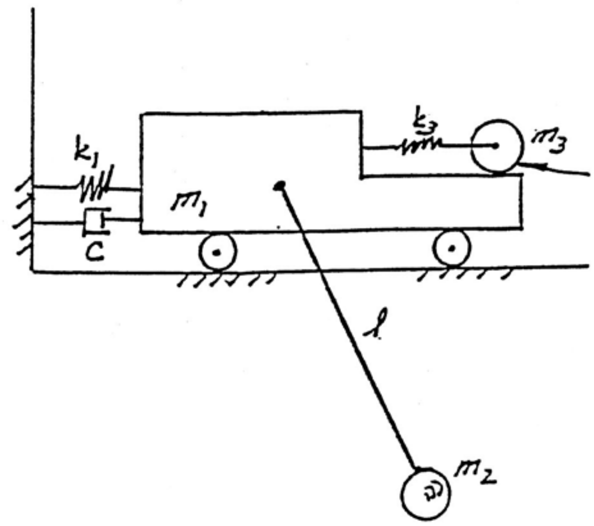
$I = I_G + md^2$

Problem #3.

Part (a). Determine the equations of motion for the system shown.

Part (b). Give a detailed outline of the steps to be performed to determine the natural frequencies and corresponding amplitudes of the system. No calculations need be shown. Neglect friction. Rod is massless. (m_2 is of negligible radius.)

$$(d/dt)(\partial T/\dot{q}_j) - \partial T/q_j + \partial D/\dot{q}_j + \partial V/q_j = Q_j$$



drum of radius r. $I_G = \frac{1}{2} mr^2$

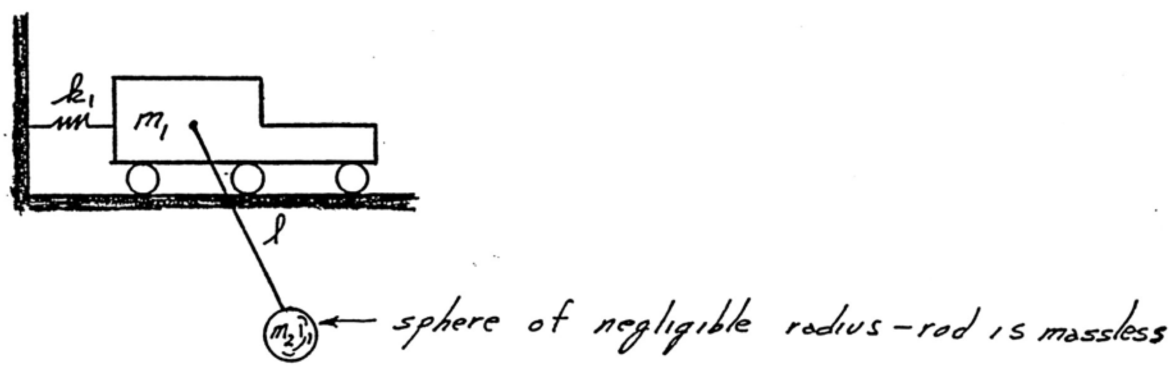
Qualifying Exam Spring 1997
Vibrations

This portion of the qualifying exam is **closed** book. Note: During this time the exam was a vibrations and controls exam, so the controls problems are not included.

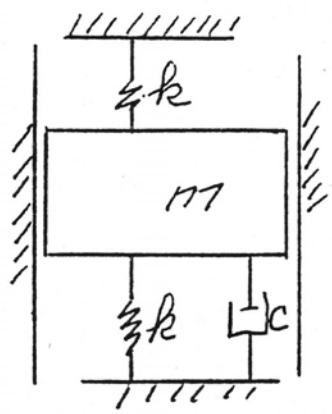
There are 4 problems here. Pick any 3 to work.

Problem #1

For the system shown determine
(a) the equations of motion, and
(b) the natural frequencies and relative amplitudes.



Problem #2   Determine the equation of motion for the system shown. Find an expression for the critical damping coefficient.



Qualifying Exam Fall 1996
Vibrations

This portion of the qualifying exam is **closed** book. Note: During this time the exam was a vibrations and controls exam, so the controls problems are not included.

There are 4 problems here. Pick any 3 to work.

Problem 1. A weight attached to a spring of stiffness 3.0 lb/in has a viscous damping device. When the weight was displaced and released the period of vibration was found to be 1.80 seconds, and the ratio of consecutive amplitudes was 4.2 to 1.0. Find the following.

(a) The logarithmic decrement.

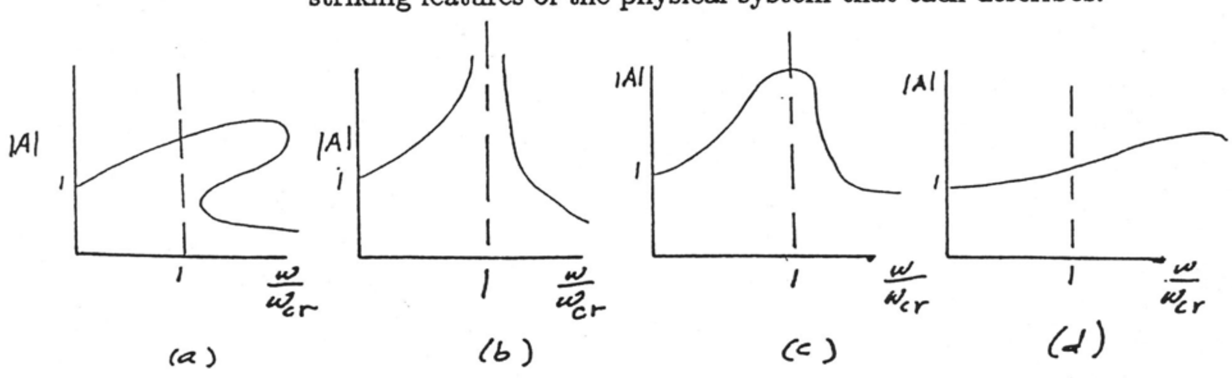
(b) The viscous damping factor.

(c) the mass of the attached weight.

Note: $\delta = \ln(x_m/x_{m+1}) = 2\pi\zeta/\sqrt{1-\zeta^2}$, $\zeta = c/c_{cr}$, $\omega_d = \sqrt{1-\zeta^2}\omega$

Problem 2. Consider parts (a) and (b) below,

(a) The following response curves are for a one degree of freedom system. Briefly (one or two sentences) describe the most striking features of the physical system that each describes.



(b) Briefly (one or two sentences) define:

i. generalized coordinates

ii. degree of freedom

iii. beating

iv. critical damping