# Dynamics Qualifying Exam
# Study Material

The candidate is expected to have a thorough understanding of engineering dynamics topics. These topics are listed below for clarification. Not all instructors cover exactly the same material during a course, thus it is important for the candidate to closely examine the subject areas listed below. The textbook listed below is a good source for the review and study of a majority of the listed topics. One final note, the example problems made available to the candidates are from past exams and do not cover all subject material. These problems are not to be used as the only source of study material. The topics listed below should be your guide for what you are responsible for knowing.

Suggested textbook:

*Dynamics*, Ginsberg & Genin, (John Wiley & Sons, pub)
*Engineering Mechanics, Dynamics,* R.C. Hibbeler, (Prentice Hall)

Topic areas:

1. Kinematics and kinetics of particles
2. Kinematics and kinetics of rigid bodies in planar motion
3. Kinematics of rigid bodies in three dimensions

**ME 237 Topics, and Assignments** (Engineering Mechanics, Dynamics, R.C. Hibbeler)

| Topic | Reading | Problems | Topic | Reading | Problems |
|---|---|---|---|---|---|
| Kinematics of a Particle | 3-14, 30-41,48-54 | 12-22, 12-85, 12-119 | Kinematics of a Particle | 61-69, 76-86 | 12-160, 12-174, 12-196 |
| Equations of Motion (Rectangular Coordinates) | 97-112 | 13-2, 13-22, 13-29 | Equations of Motion (Curvilinear Motion) | 123-128 | 13-56, 13-68, 13-70 |
| Equations of Motion (Cylindrical Coordinates) | 135-139 | 13-90, 13-98, 13-101 | Work and Energy | 159-172, 182-185 190-198 | 14-7, 14-34, 14-41 |
| Work and Energy | 159-172, 182-185 190-198 | 14-54, 14-79, 14-92 | Impulse-Momentum | 207-214, 222-228 | 15-27, 15-33, 15-51 |
| Central Impact | 233-239 | 15-71, 15-77, 15-86 | Angular Impulse-Momentum | 246-255 | 15-96, 15-101, 15-107 |
| Kinematics of Rigid Bodies | 289-299 | 16-3, 16-12, 16-23 | Kinematics of Rigid Bodies | 307-310 | 16-35, 16-40, 16-46 |
| Kinematics of Rigid Bodies | 315-322 | 16-55, 16-70, 16-77 | Kinematics of Rigid Bodies | 339-347 | 16-108, 16-117, 16-123 |
| Moving Reference Frame | 354-362 | 16-131, 16-132, 16-140 | Moment of Inertia | 371-379 | 17-2, 17-8, 17-13 |
| Equations of Motion | 385-394 | 17-27, 17-33, 17-40 | Equations of Motion | 400-407 | 17-61, 17-66, 17-73 |
| Equations of Motion | 416-422 | 17-93, 17-100, 17-110 | Work Energy | 431-445 | 18-10, 18-23,18-31 |
| Work Energy | 453-458 | 18-48, 18-53, 18-58 | Work Energy | 453-458 | 18-47, 18-50 |
| Impulse-Momentum | 465-478 | 19-4, 19-17, 19-19 | Impulse-Momentum | 486-493 | 19-37,19-43, 19-50 |
| Impulse-Momentum | 486-493 | R2-6, R2-13, R2-40 | Moving Reference Frame | 515-524 | 20-3, 20-8, 20-12 |
| Moving Reference Frame | 515-524 | 20-16, 20-18, 20-23 | Moving Reference Frame | 532-539 | 20-37, 20-43, 20-46 |
| Angular Momentum | 555-558 | 21-29, 21-30 | Angular Momentum | 555-558 | |
| Equations of Motion | 566-575 | | Work Energy | 558-559 | |
| Impulse-Momentum | 558 | | Vibrations | 595-603 | |
| Vibrations | 595-603 | 22-9, 22-17, 22-25 | | | |

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## Qualifying Exam: Dynamics

# CLOSED BOOK

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

| | | | |
|---|---|---|---|
| **Path Variables:** | $\bar{v} = v\hat{e}_t$ | | $\bar{a} = \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$ |
| **Rectangular Coordinates Constant Acceleration:** | $v = v_0 + at$ | $s = s_0 + v_0 t + \frac{1}{2}at^2$ | $v^2 - v_0^2 = 2(s - s_0)$ |
| **Cylindrical Coordinates:** | $\bar{r} = r\hat{e}_r + z\hat{e}_z$<br>$\bar{a} = (\ddot{r} - r\,\dot{\theta}^2)\hat{e}_r + (2\dot{r}\,\dot{\theta} + r\,\ddot{\theta})\hat{e}_\theta + \ddot{z}\hat{e}_z$ | | $\bar{v} = \dot{r}\hat{e}_r + r\hat{e}_\theta + \dot{z}\hat{e}_z$ |
| **2-D Equations of Motion:** | $\sum \bar{F} = m\bar{a}_G$ | $\sum \bar{M}_G = I_G\bar{\alpha}$ | $\sum(\bar{r}_i \times \bar{F}_i) + \sum \bar{M}_i = \bar{r}_G \times m\bar{a}_G + I_G\bar{\alpha}$ |
| **3-D Equations of Motion:** | $\sum \bar{F} = m\bar{a}_G$<br>$\left(\dot{\bar{H}}_A\right)_{xyz} = (I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z)\hat{\imath} + (I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z)\hat{\jmath} + (I_{zz}\alpha_z - I_{xz}\alpha_x - I_{yz}\alpha_y)\hat{k}$<br>$\bar{\boldsymbol{\omega}} = \bar{\boldsymbol{\Omega}}$<br>$\sum M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$<br>$\sum M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$<br>$\sum M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y$ | $\sum \bar{M}_A = \dot{\bar{H}}_A$ | $\dot{\bar{H}}_A = \left(\dot{\bar{H}}_A\right)_{xyz} + \bar{\omega} \times \bar{H}_A$<br>$\bar{\boldsymbol{\omega}} \neq \bar{\boldsymbol{\Omega}}$<br>$\sum M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$<br>$\sum M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x$<br>$\sum M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y$ |
| **Work – Energy:** | $\Delta T + \Delta V = (T_2 - T_1) + (V_2 - V_1) = U_{1\to2}^{(nc)}$<br>$T = \frac{1}{2}m(\bar{v}_G \cdot \bar{v}_G) + \frac{1}{2}I_G\omega^2$ | | $U_{1\to2} = \sum \int_{r_1}^{r_2}(\bar{F} \cdot d\bar{r}) = \sum \int_{\theta_1}^{\theta_2}(\bar{M} \cdot d\bar{\theta})$<br>$V_{gravity} = mgy$ $\quad V_{spring} = \frac{1}{2}k\delta^2$ |
| **Power:** | $P = \bar{F} \cdot \bar{v}$ | | $\varepsilon = efficiency = \frac{power\ output}{power\ input}$ |
| **Impulse – Momentum:** | $\sum \int \bar{F}\,dt = \Delta\, m\bar{v}_G = (m\bar{v}_G)_2 - (m\bar{v}_G)_1$<br>$e = coef\ of\ restitution = \frac{(v_B)_2 - (v_A)_1}{(v_A)_1 - (v_B)_2}$ | | $\sum \int \bar{M}_A\,dt = \Delta\bar{H}_A = (\bar{H}_A)_2 - (\bar{H}_A)_1$<br>$\bar{H}_G = I_G\bar{\omega}$ $\quad \bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$ |

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# CLOSED BOOK

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|---|---|
| **Relative Motion Equations for a Rigid Body:** | $\bar{v}_A = \bar{v}_B + \left(\bar{v}_{A/B}\right)_{rel} = \bar{v}_B + \bar{\omega}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel}$ <br> $\bar{a}_A = \bar{a}_B + \left(\bar{a}_{A/B}\right)_{rel} = \bar{a}_B + \bar{\alpha}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel} + \bar{\omega}_{AB} \times (\bar{\omega}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel})$ <br> $\bar{a}_A = \ddot{\bar{R}} + \frac{\partial^2 \bar{\rho}_A}{\partial t^2} + 2\bar{\omega} \times \frac{\partial \rho_A}{\partial t} + \dot{\bar{\omega}} \times \bar{\rho}_A + \bar{\omega} \times (\bar{\omega} \times \bar{\rho}_A)$ <br> $T = \frac{1}{2} m(\bar{v}_G \cdot \bar{v}_G) + \frac{1}{2}\bar{\omega} \cdot \bar{H}_G$ <br> $I_{xx} = (I_{x'x'})_G + m\left(y_G^2 + z_G^2\right)$ $\quad I_{xy} = \left(I_{x'y'}\right)_G + m x_G y_G$ <br> $I_{yy} = \left(I_{y'y'}\right)_G + m(x_G^2 + z_G^2)$ $\quad I_{yz} = \left(I_{y'z'}\right)_G + m y_G z_G$ <br> $I_{zz} = (I_{z'z'})_G + m\left(x_G^2 + y_G^2\right)$ $\quad I_{zx} = (I_{z'x'})_G + m z_G x_G$ <br> $\bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$ <br> $\bar{H}_A = \left(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right)\hat{\imath} + \left(I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z\right)\hat{\jmath} + \left(I_{zz}\omega_z - I_{xz}\omega_x - I_{yz}\omega_y\right)\hat{k}$ |
| **Lagrange's Equations:** | $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} + \frac{\partial V}{\partial q_j} = Q_j$ $\quad \delta U = \sum Q_j \delta q_j$ |
| **Lagrange Multipliers:** | $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = \sum \lambda_i a_{ij} + Q_j$ <br> $\sum a_{jk}\dot{q}_k + b_j = 0$ |
| **Hamilton's Canonical Equations:** | $H = \sum p_i \dot{q}_i - L$ $\quad p_n = \frac{\partial T}{\partial \dot{q}_n} = \frac{\partial L}{\partial \dot{q}_n}$ <br> $\dot{q}_i = \frac{\partial H}{\partial p_i}$ $\quad \dot{p}_i = -\frac{\partial H}{\partial q_i} + Q_i$ |

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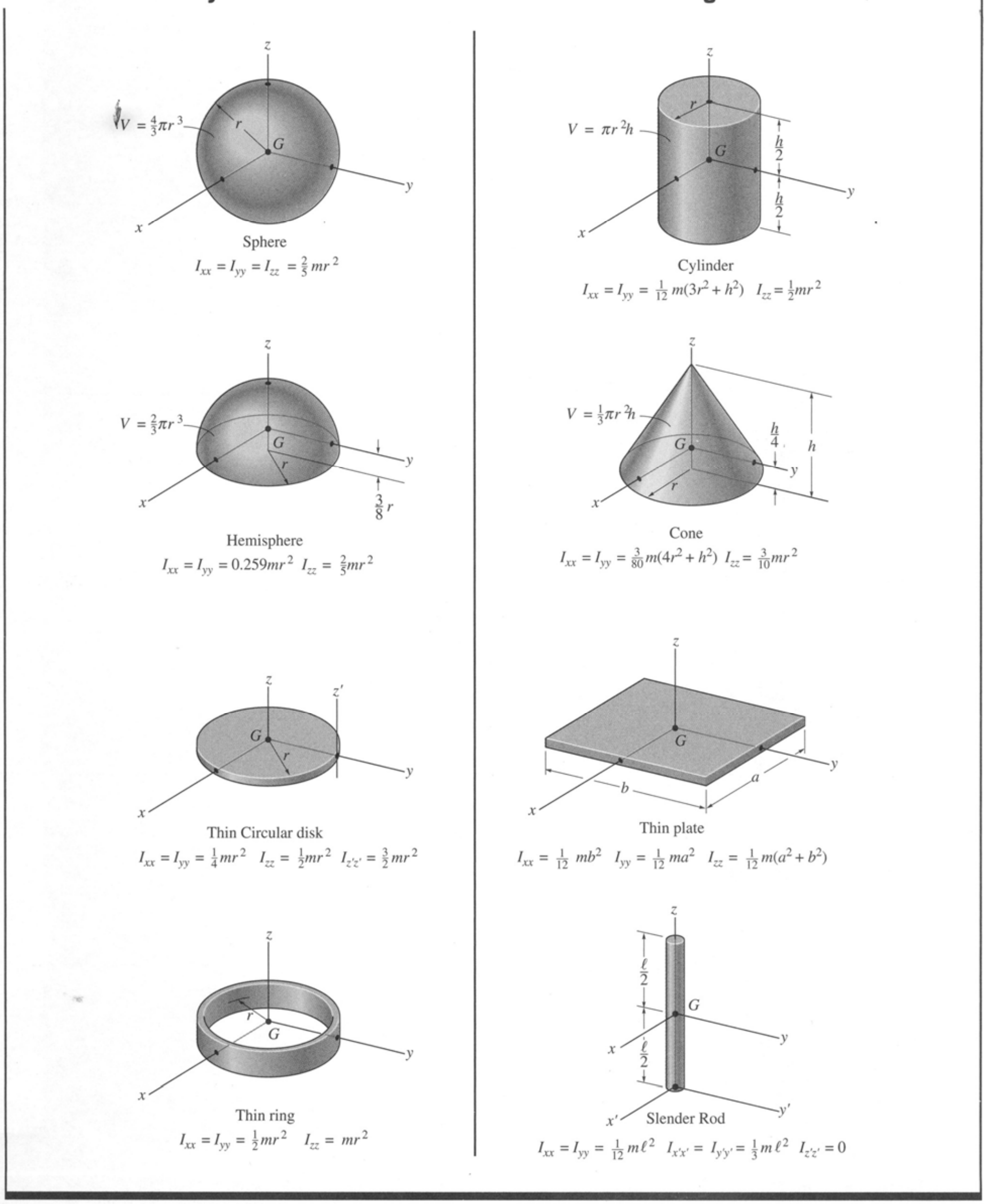
**Qualifying Exam: Dynamics**

# CLOSED BOOK

## Center of Gravity and Mass Moment of Inertia of Homogeneous Solids




Sphere
$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$

Cylinder
$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} mr^2$

Hemisphere
$I_{xx} = I_{yy} = 0.259 mr^2 \quad I_{zz} = \frac{2}{5} mr^2$

Cone
$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10} mr^2$

Thin Circular disk
$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2} mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$

Thin plate
$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$

Thin ring
$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$

Slender Rod
$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

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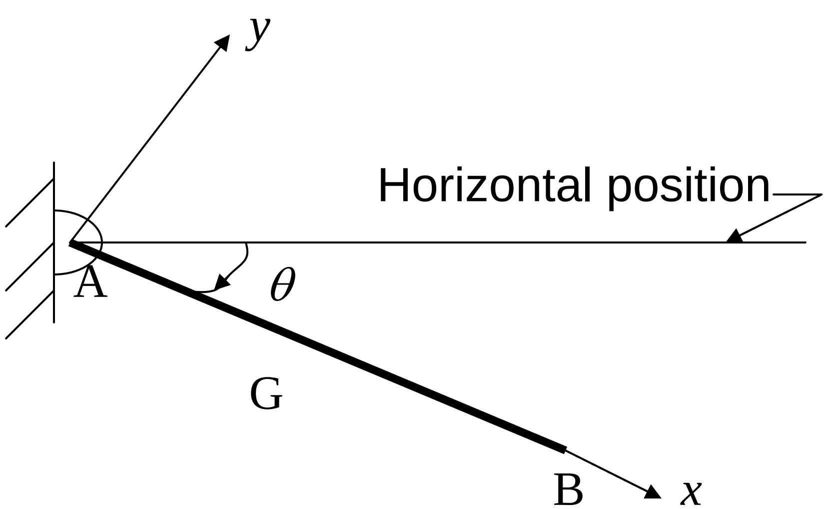
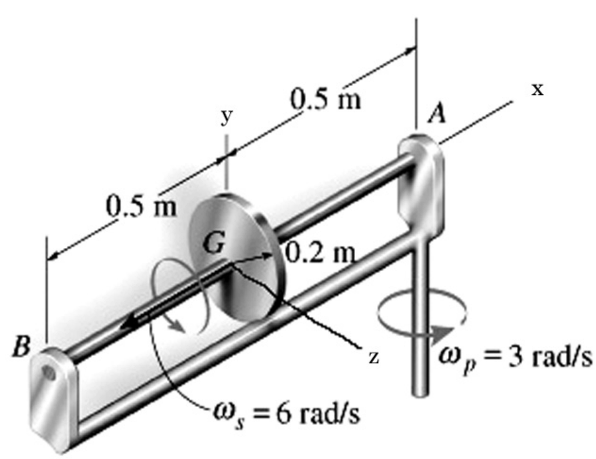
1. The slender rod AB is hinged at pin A. The mass and the length of the rod are $m$ and $L$ respectively. Point G is the mass center of the rod. It is released from the horizontal position. Using the energy method and neglecting friction, determine the velocity of point B when rod AB makes angle $\theta$ with the horizon.




2. The 10 kg flywheel (or thin disk) shown below rotates about the shaft at a constant angular velocity $\omega_s = 6$ rad/s. At the same time, the shaft is rotating about the bearing at A with a constant angular velocity of $\omega_p = 3$ rad/s. If A is a thrust bearing (forces in all 3 directions) and B is a journal bearing(no force in the x-direction), determine the components of force reaction at A and B.

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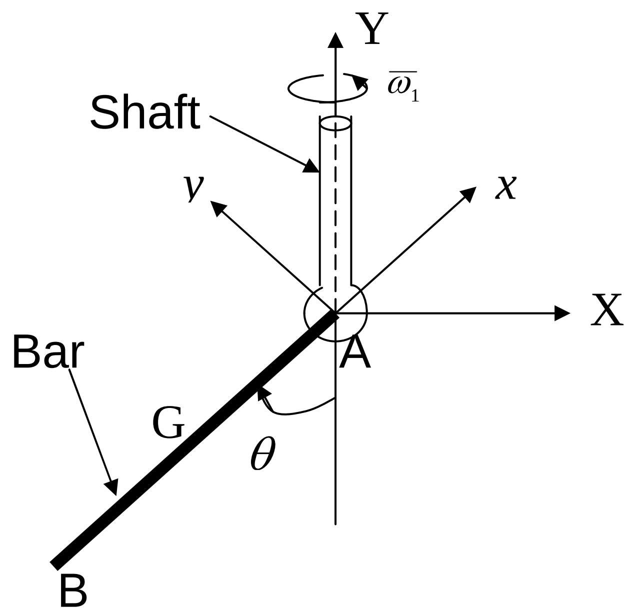
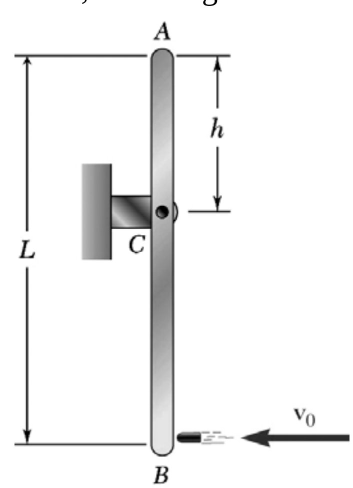
## CLOSED BOOK

3. The conical pendulum consists of bar AB of mass *m* and length *L*. It is pin-joined to a supporting shaft at point A as shown. Point G is the mass center of the bar. The shaft rotates at a constant angular velocity of $\overline{\omega}_1$. Using the Euler equation of motion, determine the angle $\theta$ that the bar makes with the vertical as it rotates. Coordinate system XYZ is fixed in space and coordinate system *xyz* is 'welded' to the bar. Assume that bar AB is a slender rod.




4. A 40-gram bullet is fired with a horizontal velocity of 600 m/s into the lower end of a slender 7-kg bar of length $L$ = 600mm. Knowing that $h$ = 240mm and that the bar is initially at rest, determine

   a. the angular velocity of the bar immediately after the bullet becomes embedded,
   b. the impulse reaction at $C$, assuming that the bullet becomes embedded in 0.001s.



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**Qualifying Exam: Dynamics**

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## Qualifying Exam: Dynamics

# CLOSED BOOK

| | | |
|---|---|---|
| **Path Variables:** | $\bar{v} = v\hat{e}_t$ | $\bar{a} = \ddot{s}\,\hat{e}_t + \dfrac{\dot{s}^2}{\rho}\hat{e}_n$ |
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| **Cylindrical Coordinates:** | $\bar{r} = r\hat{e}_r + z\hat{e}_z$ <br> $\bar{a} = \left(\ddot{r} - r\,\dot{\theta}^2\right)\hat{e}_r + \left(2\dot{r}\,\dot{\theta} + r\,\ddot{\theta}\right)\hat{e}_\theta + \ddot{z}\hat{e}_z$ | $\bar{v} = \dot{r}\hat{e}_r + r\hat{e}_\theta + \dot{z}\hat{e}_z$ |
| **2-D Equations of Motion:** | $\sum \bar{F} = m\bar{a}_G$    $\sum \bar{M}_G = I_G\bar{\alpha}$ | $\sum(\bar{r}_i \times \bar{F}_i) + \sum \bar{M}_i = \bar{r}_G \times m\bar{a}_G + I_G\bar{\alpha}$ |
| **3-D Equations of Motion:** | $\sum \bar{F} = m\bar{a}_G$    $\sum \bar{M}_A = \dot{\bar{H}}_A$ <br> $\left(\dot{\bar{H}}_A\right)_{xyz} = \left(I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z\right)\hat{\imath} + \left(I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z\right)\hat{\jmath} + \left(I_{zz}\alpha_z - I_{xz}\alpha_x - I_{yz}\alpha_y\right)\hat{k}$ <br> $\boldsymbol{\bar{\omega} = \bar{\Omega}}$ <br> $\sum M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z$ <br> $\sum M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$ <br> $\sum M_z = I_z\dot{\omega}_z - \left(I_x - I_y\right)\omega_x\omega_y$ | $\dot{\bar{H}}_A = \left(\dot{\bar{H}}_A\right)_{xyz} + \bar{\omega} \times \bar{H}_A$ <br> $\boldsymbol{\bar{\omega} \neq \bar{\Omega}}$ <br> $\sum M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$ <br> $\sum M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x$ <br> $\sum M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y$ |
| **Work – Energy:** | $\Delta T + \Delta V = (T_2 - T_1) + (V_2 - V_1) = U_{1\to2}^{(nc)}$ <br> $T = \dfrac{1}{2}\,m(\bar{v}_G \cdot \bar{v}_G) + \dfrac{1}{2}\,I_G\omega^2$ | $U_{1\to2} = \sum \int_{r_1}^{r_2} (\bar{F} \cdot d\bar{r}) = \sum \int_{\theta_1}^{\theta_2} (\bar{M} \cdot d\bar{\theta})$ <br> $V_{gravity} = mgy$    $V_{spring} = \dfrac{1}{2}k\delta^2$ |
| **Power:** | $P = \bar{F} \cdot \bar{v}$ | $\varepsilon = efficiency = \dfrac{power\ output}{power\ input}$ |
| **Impulse – Momentum:** | $\sum \int \bar{F}\,dt = \Delta\, m\bar{v}_G = (m\bar{v}_G)_2 - (m\bar{v}_G)_1$ <br> $e = coef\ of\ restitution = \dfrac{(v_B)_2 - (v_A)_1}{(v_A)_1 - (v_B)_2}$ | $\sum \int \bar{M}_A\,dt = \Delta\,\bar{H}_A = (\bar{H}_A)_2 - (\bar{H}_A)_1$ <br> $\bar{H}_G = I_G\bar{\omega}$    $\bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$ |

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| | |
|---|---|
| **Relative Motion Equations for a Rigid Body:** | $\bar{v}_A = \bar{v}_B + (\bar{v}_{A/B})_{rel} = \bar{v}_B + \bar{\omega}_{AB} \times (\bar{r}_{A/B})_{rel}$<br>$\bar{a}_A = \bar{a}_B + (\bar{a}_{A/B})_{rel} = \bar{a}_B + \bar{\alpha}_{AB} \times (\bar{r}_{A/B})_{rel} + \bar{\omega}_{AB} \times (\bar{\omega}_{AB} \times (\bar{r}_{A/B})_{rel})$<br>$\bar{a}_A = \ddot{\bar{R}} + \frac{\partial^2 \bar{\rho}_A}{\partial t^2} + 2\bar{\omega} \times \frac{\partial \rho_A}{\partial t} + \dot{\bar{\omega}} \times \bar{\rho}_A + \bar{\omega} \times (\bar{\omega} \times \bar{\rho}_A)$<br>$T = \frac{1}{2} m(\bar{v}_G \cdot \bar{v}_G) + \frac{1}{2}\bar{\omega} \cdot \bar{H}_G$<br>$I_{xx} = (I_{x'x'})_G + m(y_G^2 + z_G^2)$    $I_{xy} = (I_{x'y'})_G + m x_G y_G$<br>$I_{yy} = (I_{y'y'})_G + m(x_G^2 + z_G^2)$    $I_{yz} = (I_{y'z'})_G + m y_G z_G$<br>$I_{zz} = (I_{z'z'})_G + m(x_G^2 + y_G^2)$    $I_{zx} = (I_{z'x'})_G + m z_G x_G$<br>$\bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$<br>$\bar{H}_A = (I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z)\hat{i} + (I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z)\hat{j} + (I_{zz}\omega_z - I_{xz}\omega_x - I_{yz}\omega_y)\hat{k}$ |
| **Lagrange's Equations:** | $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} + \frac{\partial V}{\partial q_j} = Q_j$    $\delta U = \sum Q_j \delta q_j$ |
| **Lagrange Multipliers:** | $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = \sum \lambda_i a_{ij} + Q_j$<br>$\sum a_{jk}\dot{q}_k + b_j = 0$ |
| **Hamilton's Canonical Equations:** | $H = \sum p_i \dot{q}_i - L$    $p_n = \frac{\partial T}{\partial \dot{q}_n} = \frac{\partial L}{\partial \dot{q}_n}$<br>$\dot{q}_i = \frac{\partial H}{\partial p_i}$    $\dot{p}_i = -\frac{\partial H}{\partial q_i} + Q_i$ |

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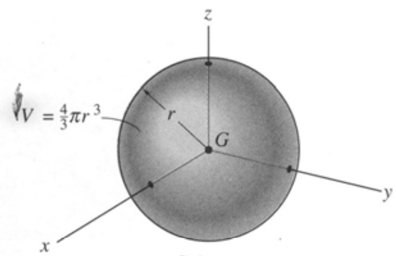
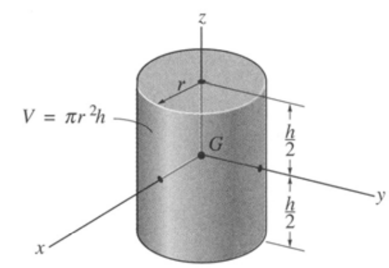
## CLOSED BOOK

### Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



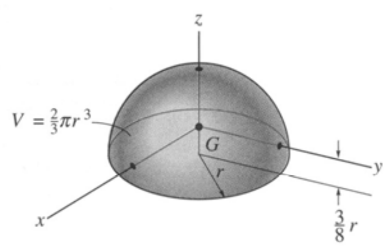

Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



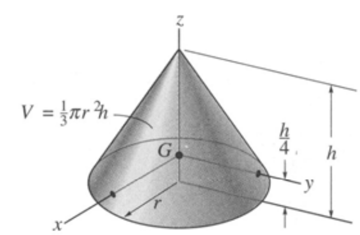

Cylinder

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} mr^2$



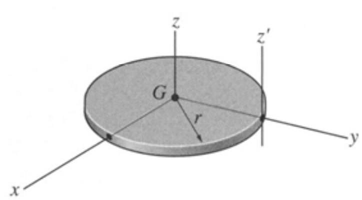

Hemisphere

$I_{xx} = I_{yy} = 0.259 mr^2 \quad I_{zz} = \frac{2}{5} mr^2$



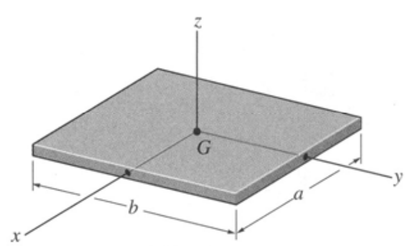

Cone

$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10} mr^2$



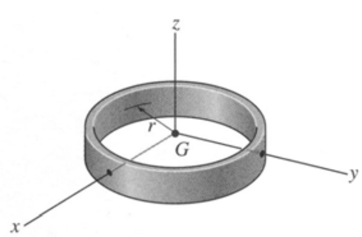

Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2} mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



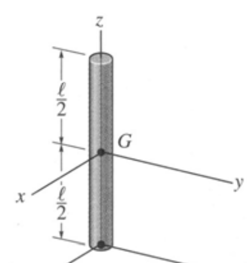

Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$




Thin ring

$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$




Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

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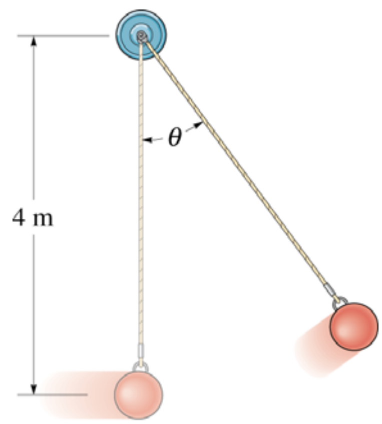
## Qualifying Exam: Dynamics

# CLOSED BOOK

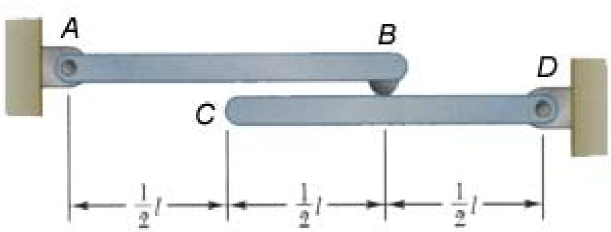
1. If the ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point (when $\theta = 0$). Neglect the size of the ball.

   1.1 Determine the tension in the cord at the instant when $\theta = 0$.

   1.2 Determine the angle $\theta$ to which the ball swings at the instant it momentarily stops.




**2.** Two slender rods, each of length $l$ and mass $m$, are released from rest in the position shown. Knowing that a small frictionless knob at end B of rod AB bears on rod CD, determine immediately after release, the acceleration of end C of rod CD and the force exerted on knob B.

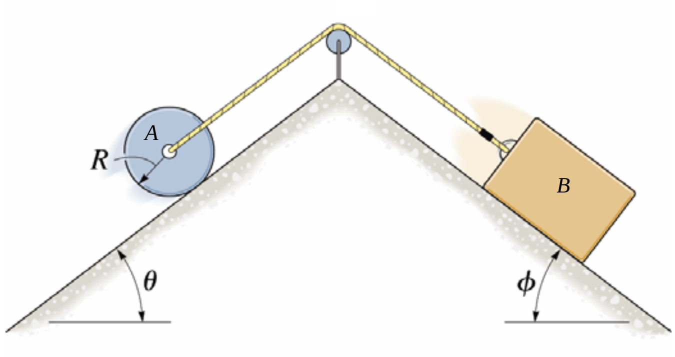


**Name ______________________________** **Summer 2007**

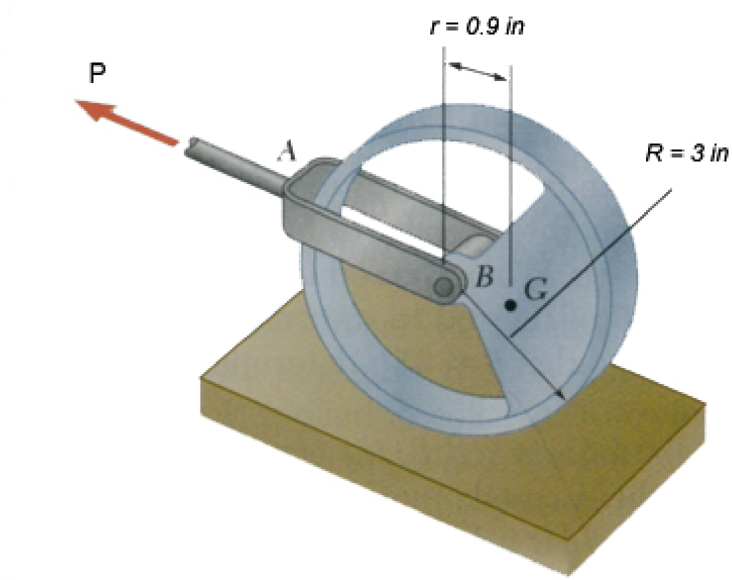
**Qualifying Exam: Dynamics**

# CLOSED BOOK

3. A cylinder and a block are connected through a rope passing over a smooth pulley as shown in the figure. The cylinder and the block have the same mass of 10 kg. Assume the cylinder rolls without slipping and the block slides without friction. Determine the acceleration of the block.




4. The center of gravity G of the unbalanced tracking wheel is located at a distance r = 0.9 inches from its geometric center B. The radius of the wheel is R = 3 inches and its centroidal radius of gyration is 2.2 inches. At the instant shown the center B of the wheel has a velocity of 1.05 ft/s and is accelerating at 3.6 ft/s$^2$, both directed to the left. Knowing that the wheel rolls without slipping and neglecting the mass of the driving yoke AB, determine the horizontal force **P** applied to the yoke.



**Name** ____________________________ **Spring 2007**

**Qualifying Exam: Dynamics**

# CLOSED BOOK

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

**Name** ______________________________ **Spring 2007**

## Qualifying Exam: Dynamics

# CLOSED BOOK

| | | | |
|---|---|---|---|
| **Path Variables:** | $\bar{v} = v\hat{e}_t$ | | $\bar{a} = \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\,\hat{e}_n$ |
| **Rectangular Coordinates Constant Acceleration:** | $v = v_0 + at$ | $s = s_0 + v_0 t + \frac{1}{2}at^2$ | $v^2 - v_0^2 = 2(s - s_0)$ |
| **Cylindrical Coordinates:** | $\bar{r} = r\hat{e}_r + z\hat{e}_z$<br>$\bar{a} = \left(\ddot{r} - r\,\dot{\theta}^2\right)\hat{e}_r + \left(2\dot{r}\,\dot{\theta} + r\,\ddot{\theta}\right)\hat{e}_\theta + \ddot{z}\hat{e}_z$ | | $\bar{v} = \dot{r}\hat{e}_r + r\hat{e}_\theta + \dot{z}\hat{e}_z$ |
| **2-D Equations of Motion:** | $\sum \bar{F} = m\bar{a}_G$ | $\sum \bar{M}_G = I_G\bar{\alpha}$ | $\sum(\bar{r}_i \times \bar{F}_i) + \sum \bar{M}_i = \bar{r}_G \times m\bar{a}_G + I_G\bar{\alpha}$ |
| **3-D Equations of Motion:** | $\sum \bar{F} = m\bar{a}_G$<br>$\left(\dot{\bar{H}}_A\right)_{xyz} = \left(I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z\right)\hat{\imath} + \left(I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z\right)\hat{\jmath} + \left(I_{zz}\alpha_z - I_{xz}\alpha_x - I_{yz}\alpha_y\right)\hat{k}$<br>$\boldsymbol{\bar{\omega}} = \boldsymbol{\bar{\Omega}}$<br>$\sum M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z$<br>$\sum M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$<br>$\sum M_z = I_z\dot{\omega}_z - \left(I_x - I_y\right)\omega_x\omega_y$ | $\sum \bar{M}_A = \dot{\bar{H}}_A$ | $\dot{\bar{H}}_A = \left(\dot{\bar{H}}_A\right)_{xyz} + \bar{\omega} \times \bar{H}_A$<br>$\boldsymbol{\bar{\omega}} \neq \boldsymbol{\bar{\Omega}}$<br>$\sum M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$<br>$\sum M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x$<br>$\sum M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y$ |
| **Work – Energy:** | $\Delta T + \Delta V = (T_2 - T_1) + (V_2 - V_1) = U_{1\to2}^{(nc)}$<br>$T = \frac{1}{2}\,m(\bar{v}_G \cdot \bar{v}_G) + \frac{1}{2}\,I_G\omega^2$ | | $U_{1\to2} = \sum \int_{r_1}^{r_2} (\bar{F} \cdot d\bar{r}) = \sum \int_{\theta_1}^{\theta_2} (\bar{M} \cdot d\bar{\theta})$<br>$V_{gravity} = mgy$ $\quad V_{spring} = \frac{1}{2}k\delta^2$ |
| **Power:** | $P = \bar{F} \cdot \bar{v}$ | | $\varepsilon = efficiency = \frac{power\ output}{power\ input}$ |
| **Impulse – Momentum:** | $\sum \int \bar{F}\,dt = \Delta\,m\bar{v}_G = (m\bar{v}_G)_2 - (m\bar{v}_G)_1$<br>$e = coef\ of\ restitution = \frac{(v_B)_2 - (v_A)_1}{(v_A)_1 - (v_B)_2}$ | | $\sum \int \bar{M}_A\,dt = \Delta\overline{H}_A = (\bar{H}_A)_2 - (\bar{H}_A)_1$<br>$\bar{H}_G = I_G\bar{\omega}$ $\quad \bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$ |

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## Qualifying Exam: Dynamics

# CLOSED BOOK

| | |
|---|---|
| **Relative Motion Equations for a Rigid Body:** | $\bar{v}_A = \bar{v}_B + \left(\bar{v}_{A/B}\right)_{rel} = \bar{v}_B + \bar{\omega}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel}$<br>$\bar{a}_A = \bar{a}_B + \left(\bar{a}_{A/B}\right)_{rel} = \bar{a}_B + \bar{\alpha}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel} + \bar{\omega}_{AB} \times (\bar{\omega}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel})$<br>$\bar{a}_A = \ddot{\bar{R}} + \frac{\partial^2 \bar{\rho}_A}{\partial t^2} + 2\bar{\omega} \times \frac{\partial \rho_A}{\partial t} + \dot{\bar{\omega}} \times \bar{\rho}_A + \bar{\omega} \times (\bar{\omega} \times \bar{\rho}_A)$<br>$T = \frac{1}{2} m(\bar{v}_G \cdot \bar{v}_G) + \frac{1}{2} \bar{\omega} \cdot \bar{H}_G$<br>$I_{xx} = (I_{x'x'})_G + m\left(y_G^2 + z_G^2\right)$ $\quad I_{xy} = \left(I_{x'y'}\right)_G + m x_G y_G$<br>$I_{yy} = \left(I_{y'y'}\right)_G + m(x_G^2 + z_G^2)$ $\quad I_{yz} = \left(I_{y'z'}\right)_G + m y_G z_G$<br>$I_{zz} = (I_{z'z'})_G + m\left(x_G^2 + y_G^2\right)$ $\quad I_{zx} = (I_{z'x'})_G + m z_G x_G$<br>$\bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$<br>$\bar{H}_A = \left(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right)\hat{i} + \left(I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z\right)\hat{j} + \left(I_{zz}\omega_z - I_{xz}\omega_x - I_{yz}\omega_y\right)\hat{k}$ |
| **Lagrange's Equations:** | $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} + \frac{\partial V}{\partial q_j} = Q_j$ $\quad \delta U = \sum Q_j \delta q_j$ |
| **Lagrange Multipliers:** | $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = \sum \lambda_i a_{ij} + Q_j$<br>$\sum a_{jk}\dot{q}_k + b_j = 0$ |
| **Hamilton's Canonical Equations:** | $H = \sum p_i \dot{q}_i - L$ $\quad p_n = \frac{\partial T}{\partial \dot{q}_n} = \frac{\partial L}{\partial \dot{q}_n}$<br>$\dot{q}_i = \frac{\partial H}{\partial p_i}$ $\quad \dot{p}_i = -\frac{\partial H}{\partial q_i} + Q_i$ |

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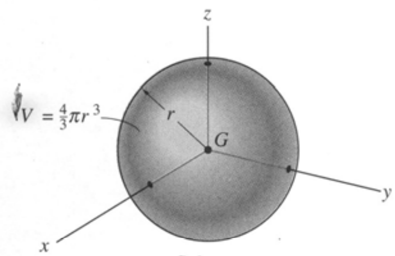
**Qualifying Exam: Dynamics**

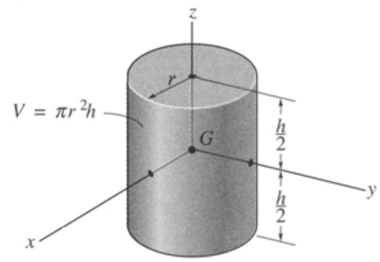
# CLOSED BOOK

## Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



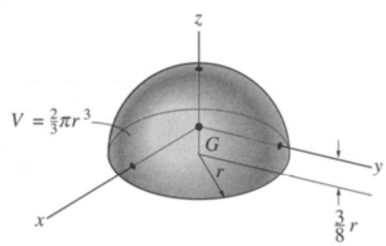

Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



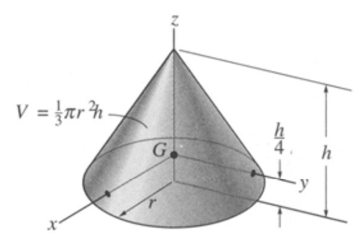

Cylinder

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} mr^2$



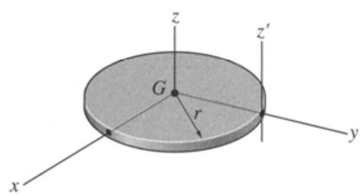

Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5}mr^2$



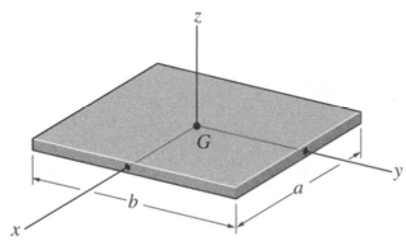

Cone

$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10} mr^2$



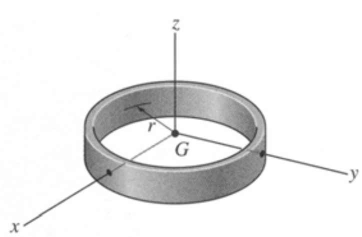
Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2} mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



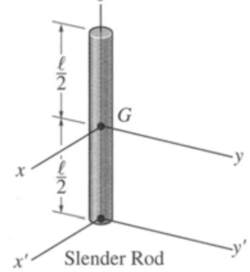
Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$



Thin ring

$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$



Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

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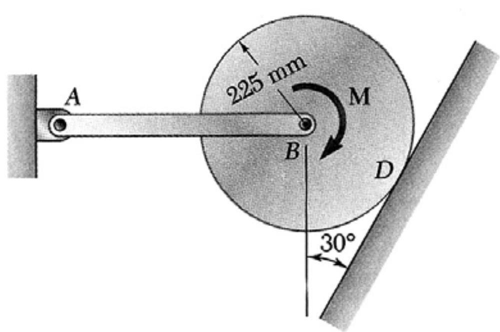
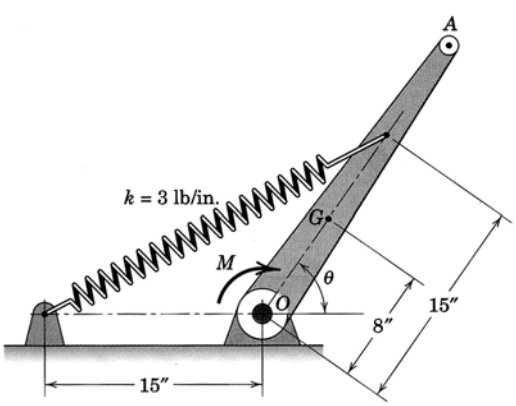
## Qualifying Exam: Dynamics

# CLOSED BOOK

1. A **10 kg** uniform disk is placed in contact with an inclined surface and a constant **11 N·m** couple **M** is applied as shown. The weight of the link **AB** is negligible. Knowing that the coefficient of kinetic friction at **D** is **0.4**, determine (a) the angular acceleration of the disk and (b) the force in link **AB**.




2. The **12 lb** lever **OA** with **10 in.** radius of gyration about **O** is initially at rest in the vertical position **(θ = 90°)**, where the attached spring of stiffness **k = 3 lb/in** is unstretched. Calculate the constant moment **M** applied to the lever through its shaft at **O** which will give the lever an angular velocity **ω = 4 rad/sec** as the lever reaches the horizontal position **θ = 0**.



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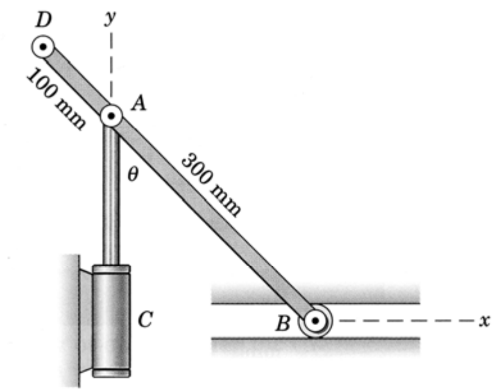
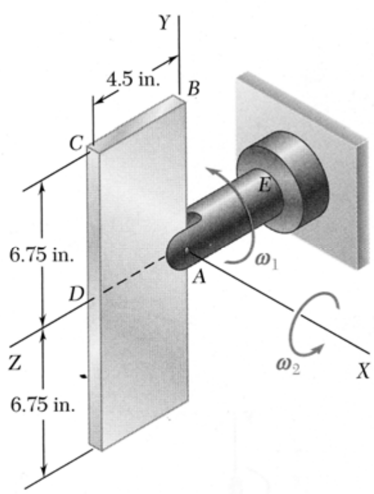
## Qualifying Exam: Dynamics

# CLOSED BOOK

3. If the piston rod of the hydraulic cylinder **C** has a constant upward velocity of **0.5 m/s**, calculate the acceleration of point **D** for the position where **$\theta$ = 45°**.




4. The rectangular plate shown rotates at the rate $\omega_2$ = 12 rad/s and is decreasing at the rate of **60 rad/s$^2$**, while the angular velocity $\omega_1$ of the arm about the **Z** axis is **9 rad/s** and is decreasing at the rate **45 rad/s$^2$**. For the position shown, determine (a) the velocity of corner **C** and (b) the acceleration of corner **C**.



**Name ______________________________** **Fall 2006**

**Qualifying Exam: Dynamics**

# CLOSED BOOK

This portion of the qualifying exam is **closed** book. You may have a calculator.

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Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

Name ______________________________ **Fall 2006**

## Qualifying Exam: Dynamics

# CLOSED BOOK

| | | |
|---|---|---|
| **Path Variables:** | $\bar{v} = v\hat{e}_t$ | $\bar{a} = \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$ |
| **Rectangular Coordinates Constant Acceleration:** | $v = v_0 + at$ $\quad s = s_0 + v_0 t + \frac{1}{2}at^2$ | $v^2 - v_0^2 = 2(s - s_0)$ |
| **Cylindrical Coordinates:** | $\bar{r} = r\hat{e}_r + z\hat{e}_z$ <br> $\bar{a} = (\ddot{r} - r\,\dot{\theta}^2)\hat{e}_r + (2\dot{r}\,\dot{\theta} + r\,\ddot{\theta})\hat{e}_\theta + \ddot{z}\hat{e}_z$ | $\bar{v} = \dot{r}\hat{e}_r + r\hat{e}_\theta + \dot{z}\hat{e}_z$ |
| **2-D Equations of Motion:** | $\sum \bar{F} = m\bar{a}_G$ $\quad \sum \bar{M}_G = I_G\bar{\alpha}$ | $\sum(\bar{r}_i \times \bar{F}_i) + \sum \bar{M}_i = \bar{r}_G \times m\bar{a}_G + I_G\bar{\alpha}$ |
| **3-D Equations of Motion:** | $\sum \bar{F} = m\bar{a}_G$ $\quad \sum \bar{M}_A = \dot{\bar{H}}_A$ <br> $(\dot{\bar{H}}_A)_{xyz} = (I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z)\hat{\imath} + (I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z)\hat{\jmath} + (I_{zz}\alpha_z - I_{xz}\alpha_x - I_{yz}\alpha_y)\hat{k}$ <br> $\boldsymbol{\bar{\omega}} = \boldsymbol{\bar{\Omega}}$ <br> $\sum M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$ <br> $\sum M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$ <br> $\sum M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y$ | $\dot{\bar{H}}_A = (\dot{\bar{H}}_A)_{xyz} + \bar{\omega} \times \bar{H}_A$ <br> $\boldsymbol{\bar{\omega}} \neq \boldsymbol{\bar{\Omega}}$ <br> $\sum M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$ <br> $\sum M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x$ <br> $\sum M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y$ |
| **Work – Energy:** | $\Delta T + \Delta V = (T_2 - T_1) + (V_2 - V_1) = U_{1\to2}^{(nc)}$ <br> $T = \frac{1}{2}\,m(\bar{v}_G \cdot \bar{v}_G) + \frac{1}{2}\,I_G\omega^2$ | $U_{1\to2} = \sum \int_{r_1}^{r_2} (\bar{F} \cdot d\bar{r}) = \sum \int_{\theta_1}^{\theta_2} (\bar{M} \cdot d\bar{\theta})$ <br> $V_{gravity} = mgy$ $\quad V_{spring} = \frac{1}{2}k\delta^2$ |
| **Power:** | $P = \bar{F} \cdot \bar{v}$ | $\varepsilon = efficiency = \frac{power\ output}{power\ input}$ |

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## Qualifying Exam: Dynamics

# CLOSED BOOK

| | | |
|---|---|---|
| **Impulse – Momentum:** | $\sum\int \bar{F}\,dt = \Delta\, m\bar{v}_G = (m\bar{v}_G)_2 - (m\bar{v}_G)_1$ <br> $e = coef\ of\ restitution = \dfrac{(v_B)_2 - (v_A)_1}{(v_A)_1 - (v_B)_2}$ | $\sum\int \bar{M}_A\,dt = \Delta\overline{H}_A = (\bar{H}_A)_2 - (\bar{H}_A)_1$ <br> $\bar{H}_G = I_G\bar{\omega}$ $\quad \bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$ |
| **Relative Motion Equations for a Rigid Body:** | $\bar{v}_A = \bar{v}_B + \left(\bar{v}_{A/B}\right)_{rel} = \bar{v}_B + \bar{\omega}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel}$ <br> $\bar{a}_A = \bar{a}_B + \left(\bar{a}_{A/B}\right)_{rel} = \bar{a}_B + \bar{\alpha}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel} + \bar{\omega}_{AB} \times (\bar{\omega}_{AB} \times \left(\bar{r}_{A/B}\right)_{rel})$ <br> $\bar{a}_A = \ddot{\bar{R}} + \dfrac{\partial^2\bar{\rho}_A}{\partial t^2} + 2\bar{\omega} \times \dfrac{\partial\rho_A}{\partial t} + \dot{\bar{\omega}} \times \bar{\rho}_A + \bar{\omega} \times (\bar{\omega} \times \bar{\rho}_A)$ <br> $T = \dfrac{1}{2}m(\bar{v}_G \cdot \bar{v}_G) + \dfrac{1}{2}\bar{\omega} \cdot \bar{H}_G$ <br> $I_{xx} = (I_{x'x'})_G + m(y_G^2 + z_G^2)$ <br> $I_{yy} = \left(I_{y'y'}\right)_G + m(x_G^2 + z_G^2)$ <br> $I_{zz} = (I_{z'z'})_G + m(x_G^2 + y_G^2)$ <br> $\bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$ <br> $\bar{H}_A = \left(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right)\hat{\imath} + \left(I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z\right)\hat{\jmath} + \left(I_{zz}\omega_z - I_{xz}\omega_x - I_{yz}\omega_y\right)\hat{k}$ | $I_{xy} = \left(I_{x'y'}\right)_G + mx_Gy_G$ <br> $I_{yz} = \left(I_{y'z'}\right)_G + my_Gz_G$ <br> $I_{zx} = (I_{z'x'})_G + mz_Gx_G$ |
| **Lagrange's Equations:** | $\dfrac{d}{dt}\left(\dfrac{\partial T}{\partial\dot{q}_j}\right) - \dfrac{\partial T}{\partial q_j} + \dfrac{\partial D}{\partial\dot{q}_j} + \dfrac{\partial V}{\partial q_j} = Q_j$ | $\delta U = \sum Q_j\delta q_j$ |
| **Lagrange Multipliers:** | $\dfrac{d}{dt}\left(\dfrac{\partial T}{\partial\dot{q}_j}\right) - \dfrac{\partial T}{\partial q_j} + \dfrac{\partial V}{\partial q_j} = \sum \lambda_i a_{ij} + Q_j$ <br> $\sum a_{jk}\dot{q}_k + b_j = 0$ | |
| **Hamilton's Canonical Equations:** | $H = \sum p_i\dot{q}_i - L$ <br> $\dot{q}_i = \dfrac{\partial H}{\partial p_i}$ | $p_n = \dfrac{\partial T}{\partial\dot{q}_n} = \dfrac{\partial L}{\partial\dot{q}_n}$ <br> $\dot{p}_i = -\dfrac{\partial H}{\partial q_i} + Q_i$ |

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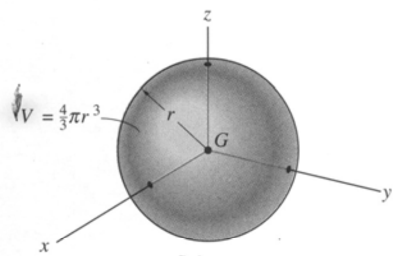
**Qualifying Exam: Dynamics**

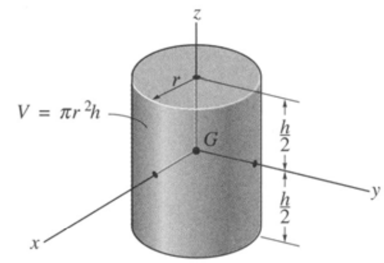
## CLOSED BOOK

### Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



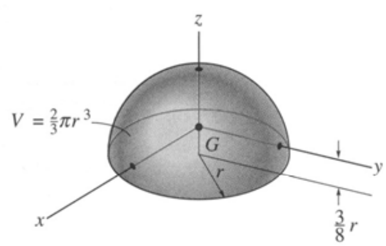

Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



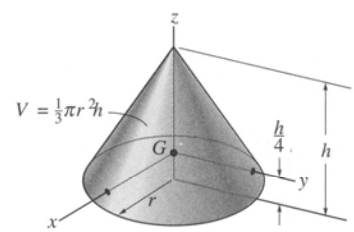

Cylinder

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} mr^2$



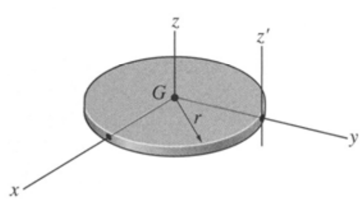

Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5} mr^2$



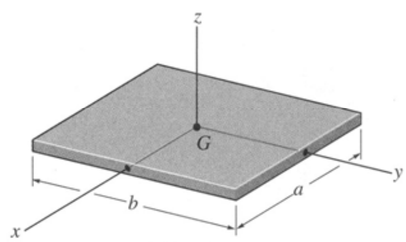

Cone

$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10} mr^2$



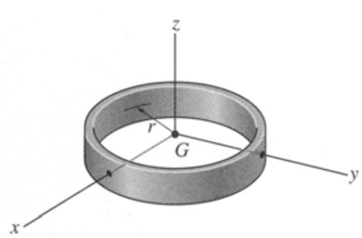

Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2} mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



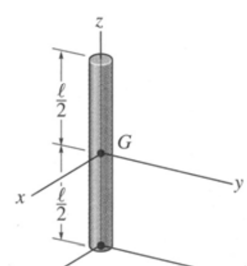

Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$




Thin ring

$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$




Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

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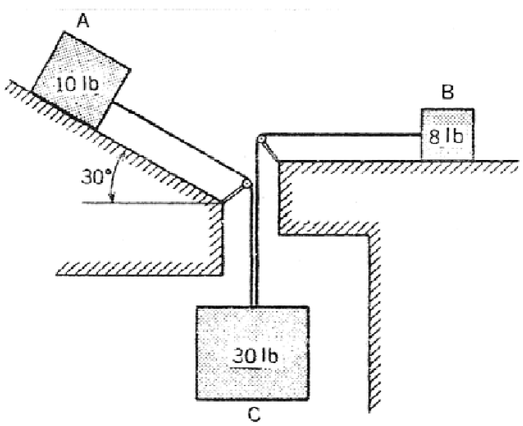
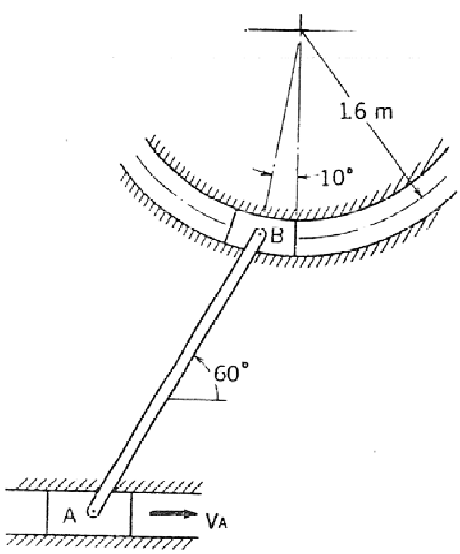
## Qualifying Exam: Dynamics

# CLOSED BOOK

1. The system shown is released from rest. What distance does the body C drop in 2 seconds? The cables are inextensible. The coefficient of dynamic friction $\mu_d$ is 0.4 for contact surfaces of bodies A and B.




2. At the instant shown slider A has a speed of 3 m/s. Member AB is 2.5 meters long. Determine the angular velocity of member AB.



**Name** ______________________________ **Fall 2006**

## Qualifying Exam: Dynamics

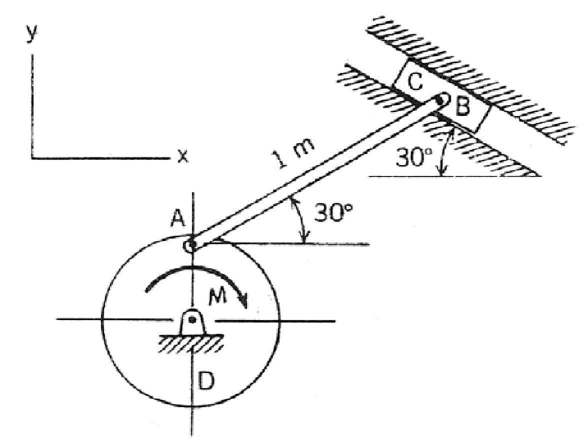
# CLOSED BOOK

3. A moment of 100 N-m is applied to a wheel D having a mass of 50 kg, a diameter of 600mm, and a radius of gyration of 280 mm. The wheel D is attached by a light member AB to a slider C having a mass of 30 kg.

   a. If the system is at rest at the instant shown, what is the acceleration of slider C?

   b. What is the axial force in member AB?

   Neglect friction everywhere, and neglect the inertia of the member AB.

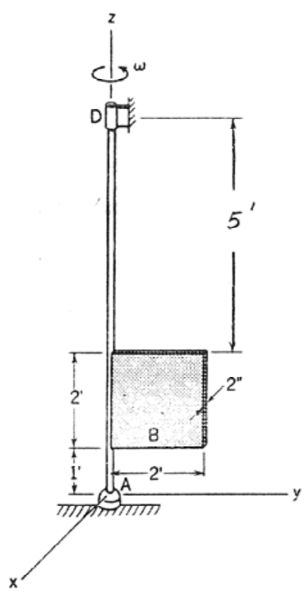


**Name ______________________________** **Fall 2006**

## Qualifying Exam: Dynamics

# CLOSED BOOK

4. A vertical shaft rotates with angular speed $\omega$ of 5 rad/s in bearings A and D. A uniform plate B weighing 50 lb is attached to the shaft as shown in the diagram below. What are the bearing reactions at the configuration shown? The shaft weighs 20 lb and the thickness of the plate is 2 in.



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**Qualifying Examination**
**Subject: Dynamics**

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

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Rectangular Coordinates: $\bar{v} = d\bar{r}/dt$ $\bar{a} = d\bar{v}/dt$

Constant Acceleration: $v = v_o + at$ $s = s_o + v_o t + \frac{1}{2}at^2$ $v^2 - v_o{}^2 = 2a(s - s_o)$

Path Variables: $\bar{v} = v\bar{u}_t$ $\bar{a} = \dot{v}\bar{u}_t + \dfrac{v^2}{\rho}\bar{u}_n$

Cylindrical Coordinates: $\bar{r} = r\bar{u}_r + z\bar{u}_z$ $\bar{v} = \dot{r}\bar{u}_r + r\dot{\theta}\bar{u}_\theta + \dot{z}\bar{u}_z$ $\bar{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\bar{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\bar{u}_\theta + \ddot{z}\bar{u}_z$

Relative Motion Equations for a rigid body:

$$\bar{v}_A = \bar{v}_B + \bar{v}_{A/B} = \bar{v}_B + \bar{\omega}_{AB} \times \bar{r}_{A/B}$$
$$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B} = \bar{a}_B + \bar{\alpha}_{AB} \times \bar{r}_{A/B} - \omega_{AB}^2 \bar{r}_{A/B}$$

Equations of Motion: $\sum \bar{F} = m\bar{a}$ $\sum M_G = I_G \alpha$ $\sum\left(\bar{r}_i \times \bar{F}_i\right) + \sum \bar{M}_i = \bar{r}_G \times m\bar{a}_G + I_G \bar{\alpha}$

Work - Energy: $T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ $T = \frac{1}{2}m\bar{v}_G \cdot \bar{v}_G + \frac{1}{2}\bar{\omega} \cdot \bar{H}_G$ $T = \frac{1}{2}\bar{\omega} \cdot \bar{H}_O$

$V_{sp} = \frac{1}{2}ks^2$ $V_{gr} = Wy$ $U_{1\to2} = \int_{r_1}^{r_2} \bar{F} \cdot d\bar{r}$ $U_{1\to2} = \int_{\theta_1}^{\theta_2} \bar{M} \cdot d\bar{\theta}$

Power: $P = \bar{F} \cdot \bar{v}$ $\varepsilon = \dfrac{power\ output}{power\ input}$

Impulse - Momentum: $m(\bar{v}_G)_2 = m(\bar{v}_G)_1 + \sum \int \bar{F} dt$ $\left(\bar{H}_O\right)_2 = \left(\bar{H}_O\right)_1 + \sum \int \bar{M}_O dt$

$e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ $H_G = I_G \omega$ $\bar{H}_o = \bar{H}_G + \bar{r}_{G/o} \times m\bar{v}_G$

Three Dimensional Dynamics:

$$I_{xx} = \left(I_{x'x'}\right)_G + m\left(y_G{}^2 + z_G{}^2\right) \qquad I_{xy} = \left(I_{x'y'}\right)_G + mx_G y_G$$
$$I_{yy} = \left(I_{y'y'}\right)_G + m\left(x_G{}^2 + z_G{}^2\right) \qquad I_{yz} = \left(I_{y'z'}\right)_G + my_G z_G$$
$$I_{zz} = \left(I_{z'z'}\right)_G + m\left(x_G{}^2 + y_G{}^2\right) \qquad I_{zx} = \left(I_{z'x'}\right)_G + mz_G x_G$$
$$\bar{H}_A = \bar{r}_{G/A} \times m\bar{v}_G + \bar{H}_G$$
$$\bar{H}_A = \left(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right)\hat{i} + \left(I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z\right)\hat{j} + \left(I_{zz}\omega_z - I_{xz}\omega_x - I_{yz}\omega_y\right)\hat{k}$$

Equations of Motion: $\sum \bar{F} = m\bar{a}_G$ $\sum \bar{M}_A = \dot{\bar{H}}_A$ $\dot{\bar{H}}_A = \left(\dot{\bar{H}}_A\right)_{xyz} + \bar{\omega} \times \bar{H}_A$

$$\left(\dot{\bar{H}}_A\right)_{xyz} = \left(I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z\right)\hat{i} + \left(I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z\right)\hat{j} + \left(I_{zz}\alpha_z - I_{xz}\alpha_x - I_{yz}\alpha_y\right)\hat{k}$$

| $\bar{\omega} = \bar{\Omega}$ | $\bar{\omega} \neq \bar{\Omega}$ |
|---|---|
| $\sum M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z$ | $\sum M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$ |
| $\sum M_y = I_y\dot{\omega}_y - \left(I_z - I_x\right)\omega_z\omega_x$ | $\sum M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x$ |
| $\sum M_z = I_z\dot{\omega}_z - \left(I_x - I_y\right)\omega_x\omega_y$ | $\sum M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y$ |

Moving reference frame:

$$\bar{v}_P = \bar{v}_A + (\bar{v}_{P/A})_{rel} + \bar{\omega} \times \bar{r}_{P/A}$$
$$\bar{a}_P = \bar{a}_A + \left(\bar{a}_{P/A}\right)_{rel} + 2\bar{\omega} \times \left(\bar{v}_{P/A}\right)_{rel} + \bar{\alpha} \times \bar{r}_{P/A} + \bar{\omega} \times \left(\bar{\omega} \times \bar{r}_{P/A}\right)$$

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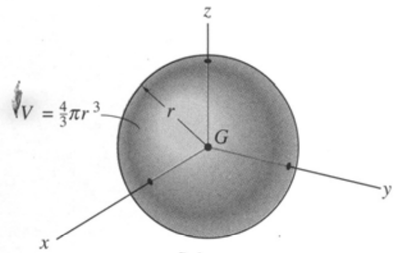
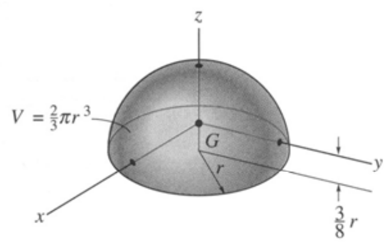
## Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



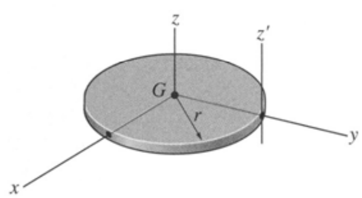

Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$



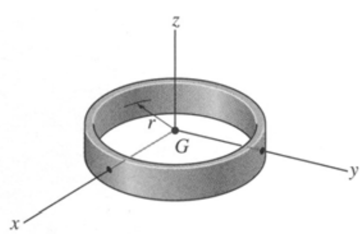

Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5}mr^2$



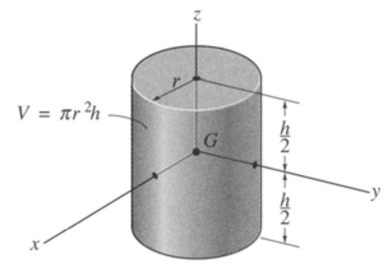

Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4}mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2}mr^2$



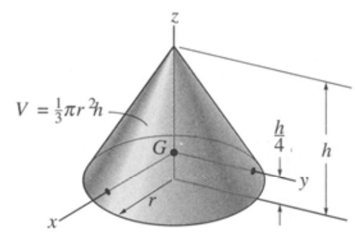

Thin ring

$I_{xx} = I_{yy} = \frac{1}{2}mr^2 \quad I_{zz} = mr^2$




Cylinder

$I_{xx} = I_{yy} = \frac{1}{12}m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2}mr^2$

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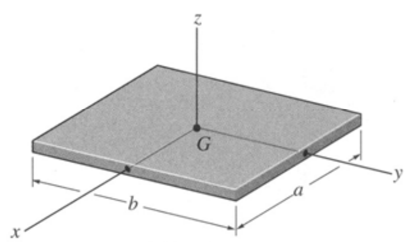
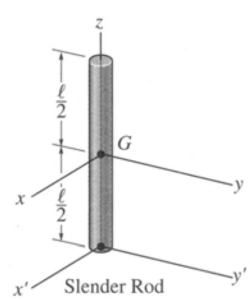



Thin plate

$I_{xx} = \frac{1}{12}mb^2 \quad I_{yy} = \frac{1}{12}ma^2 \quad I_{zz} = \frac{1}{12}m(a^2 + b^2)$




Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12}m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3}m\ell^2 \quad I_{z'z'} = 0$

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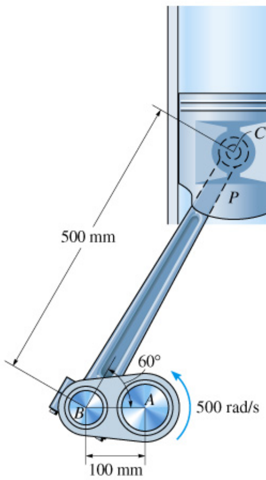
# Qualifying Examination
# Subject: Dynamics

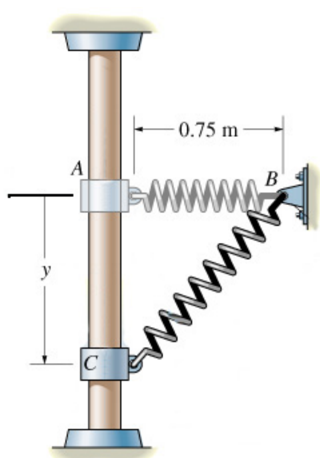
## Problem 1:

The crankshaft AB is rotating at 500 rad/s about a fixed axis passing through A. Determine the speed of the piston P at the instant it is in the position shown.




## Problem 2:

The mass of collar A is 2 kg and the spring constant is 60 N/m. The collar has no velocity at A and the spring is un-deformed at A. Determine the maximum distance y the collar drops before it stops at Point C.



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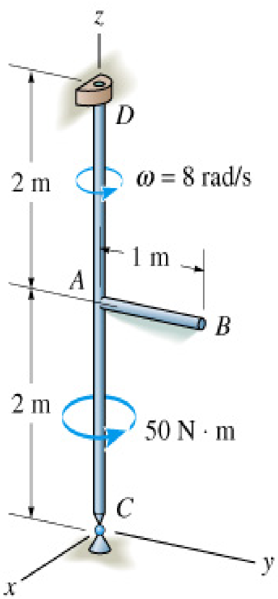
# Qualifying Examination
# Subject: Dynamics

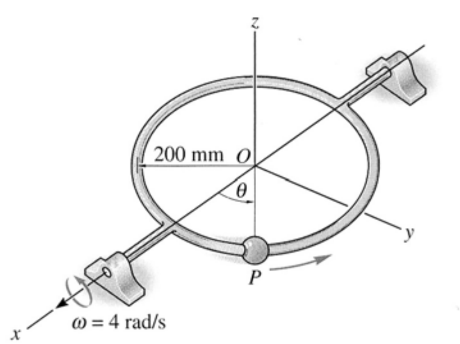
## Problem 3:

The rod assembly is supported by a ball-and-socket joint at ***C*** and a journal bearing at ***D***, which develops only *x* and *y* force reactions. The rods (CD and AB) have a mass of 0.75 kg/m each. Determine the angular acceleration of the rods and the components of the reaction at the supports at the instant ω = 8 rad/s as shown.




## Problem 4:

The particle P slides around the circular hoop with a constant angular velocity of $\dot{\theta}$ = 6 rad/sec, while the hoop rotates about the x axis at a constant rate of ω = 4 rad/s. If at the instant shown the hoop is in the x-y plane and the angle θ = 45°, determine the velocity and acceleration of the particle at this instant.



**Name ______________________________** **Fall 2005**

**Qualifying Exam: Dynamics**

# CLOSED BOOK

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**Name** ______________________________ **Fall 2005**

## Qualifying Exam: Dynamics

# CLOSED BOOK

Rectangular Coordinates:

Constant Acceleration: $v = v_o + at$ $s = s_o + v_o t + \frac{1}{2}at^2$ $v^2 - v_o{}^2 = 2a(s - s_o)$

Path Variables: $\bar{v} = v\bar{u}_t$ $\bar{a} = \dot{v}\bar{u}_t + \frac{v^2}{\rho}\bar{u}_n$

Cylindrical Coordinates: $\bar{r} = r\bar{u}_r + z\bar{u}_z$ $\bar{v} = \dot{r}\bar{u}_r + r\dot{\theta}\bar{u}_\theta + \dot{z}\bar{u}_z$ $\bar{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\bar{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\bar{u}_\theta + \ddot{z}\bar{u}_z$

Relative Motion Equations for a rigid body:

$$\bar{v}_A = \bar{v}_B + \bar{v}_{A/B} = \bar{v}_B + \bar{\omega}_{AB} \times \bar{r}_{A/B}$$

$$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B} = \bar{a}_B + \bar{\alpha}_{AB} \times \bar{r}_{A/B} - \omega_{AB}^2 \bar{r}_{A/B}$$

Equations of Motion: $\sum \bar{F} = m\bar{a}$ $\sum M_G = I_G\alpha$ $\sum\left(\bar{r}_i \times \bar{F}_i\right) + \sum \bar{M}_i = \bar{r}_G \times m\bar{a}_G + I_G\bar{\alpha}$

Work - Energy: $T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ $T = \frac{1}{2}m\bar{v}_G \cdot \bar{v}_G + \frac{1}{2}\bar{\omega}\cdot\bar{H}_G$ $T = \frac{1}{2}\bar{\omega}\cdot\bar{H}_O$

$$V_{sp} = \tfrac{1}{2}ks^2 \qquad V_{gr} = Wy \qquad U_{1\to2} = \int_{r_1}^{r_2}\bar{F}\cdot d\bar{r} \qquad U_{1\to2} = \int_{\theta_1}^{\theta_2}\bar{M}\cdot d\bar{\theta}$$

Power: $P = \bar{F}\cdot\bar{v}$ $\varepsilon = \dfrac{power\ output}{power\ input}$

Impulse - Momentum:

$$m(\bar{v}_G)_2 = m(\bar{v}_G)_1 + \sum\int \bar{F}dt \qquad \left(\bar{H}_O\right)_2 = \left(\bar{H}_O\right)_1 + \sum\int \bar{M}_O dt$$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \qquad H_G = I_G\omega \qquad \bar{H}_o = \bar{H}_G + \bar{r}_{G/o} \times m\bar{v}_G$$

Three Dimensional Dynamics:

$$I_{xx} = \left(I_{x'x'}\right)_G + m\left(y_G{}^2 + z_G{}^2\right) \qquad I_{xy} = \left(I_{x'y'}\right)_G + m x_G y_G$$

$$I_{yy} = \left(I_{y'y'}\right)_G + m\left(x_G{}^2 + z_G{}^2\right) \qquad I_{yz} = \left(I_{y'z'}\right)_G + m y_G z_G$$

$$I_{zz} = \left(I_{z'z'}\right)_G + m\left(x_G{}^2 + y_G{}^2\right) \qquad I_{zx} = \left(I_{z'x'}\right)_G + m z_G x_G$$

$$\bar{H}_A = \bar{r}_{G/A} \times m\bar{v}_G + \bar{H}_G$$

$$\bar{H}_A = \left(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right)\hat{i} + \left(I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z\right)\hat{j} + \left(I_{zz}\omega_z - I_{xz}\omega_x - I_{yz}\omega_y\right)\hat{k}$$

Equations of Motion: $\sum \bar{F} = m\bar{a}_G$ $\sum \bar{M}_A = \dot{\bar{H}}_A$ $\dot{\bar{H}}_A = \left(\dot{\bar{H}}_A\right)_{xyz} + \bar{\omega}\times\bar{H}_A$

$$\left(\dot{\bar{H}}_A\right)_{xyz} = \left(I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z\right)\hat{i} + \left(I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z\right)\hat{j} + \left(I_{zz}\alpha_z - I_{xz}\alpha_x - I_{yz}\alpha_y\right)\hat{k}$$

$$\bar{\omega} = \bar{\Omega} \qquad\qquad \bar{\omega} \neq \bar{\Omega}$$

$$\sum M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z \qquad \sum M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$$

$$\sum M_y = I_y\dot{\omega}_y - \left(I_z - I_x\right)\omega_z\omega_x \qquad \sum M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x$$

$$\sum M_z = I_z\dot{\omega}_z - \left(I_x - I_y\right)\omega_x\omega_y \qquad \sum M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y$$

Moving reference frame:

$$\bar{v}_P = \bar{v}_A + \left(\bar{v}_{P/A}\right)_{rel} + \bar{\omega}\times\bar{r}_{P/A}$$

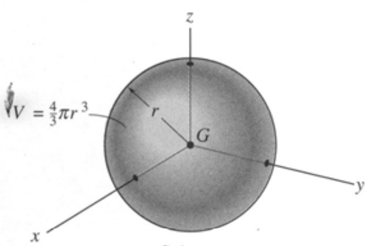
$$\bar{a}_P = \bar{a}_A + \left(\bar{a}_{P/A}\right)_{rel} + 2\bar{\omega}\times\left(\bar{v}_{P/A}\right)_{rel} + \bar{\alpha}\times\bar{r}_{P/A} + \bar{\omega}\times\left(\bar{\omega}\times\bar{r}_{P/A}\right)$$

**Name** ______________________________ **Fall 2005**

## Qualifying Exam: Dynamics

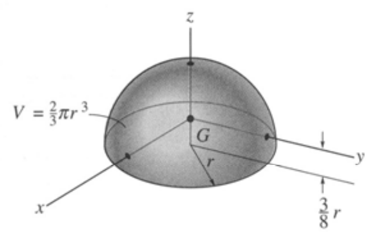
# CLOSED BOOK

### Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



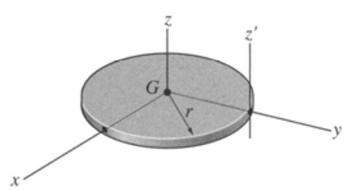

Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



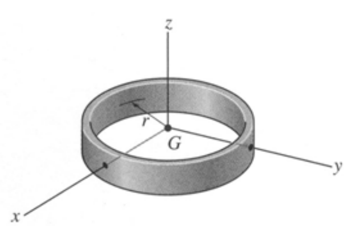

Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5}mr^2$



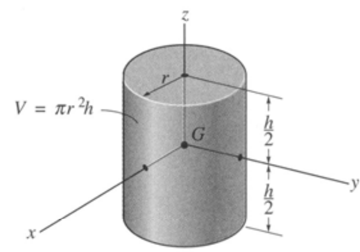

Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4}mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2}mr^2$



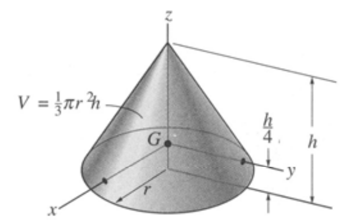

Thin ring

$I_{xx} = I_{yy} = \frac{1}{2}mr^2 \quad I_{zz} = mr^2$



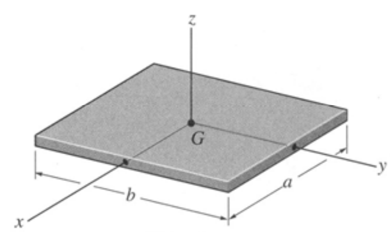

Cylinder

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2}mr^2$



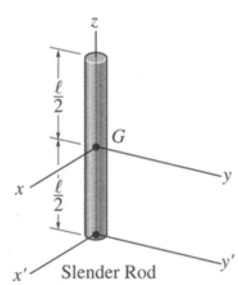

Cone

$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10}mr^2$




Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$




Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

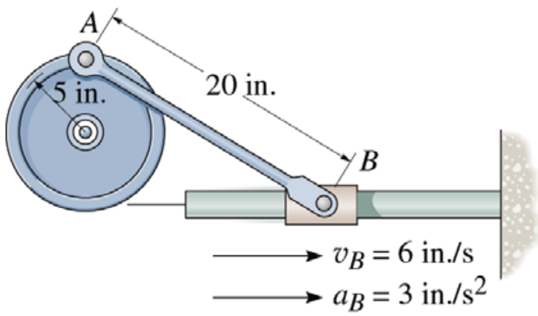
**Name** ______________________________ **Fall 2005**

## Qualifying Exam: Dynamics

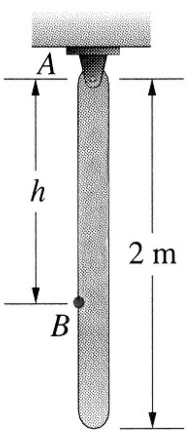
# CLOSED BOOK

1. At a given instant, the slider ***B*** is traveling to the right with the velocity and acceleration shown.

   a) Indicate where the instantaneous center of zero velocity of link ***AB*** is located.
   b) Determine the angular acceleration of the wheel at this instant.




2. The suspended **8 kg** slender bar is subjected to a horizontal impulsive force at **B**. The average value of the force is **1000 N**, and its duration is **0.03 s**. If the force causes the bar to swing to the horizontal position before coming to a stop, what is the distance h?

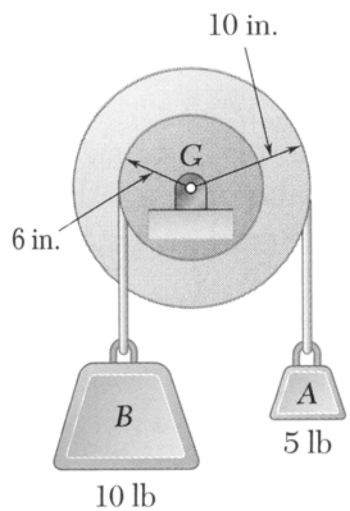


**Name ________________________________** **Fall 2005**

## Qualifying Exam: Dynamics

# CLOSED BOOK

3. A pulley weighing **12 lb** and having a radius of gyration of **8 in.** is connected to two blocks as shown. Assuming no axial friction, determine the ***angular acceleration*** of the pulley and the acceleration of each block.

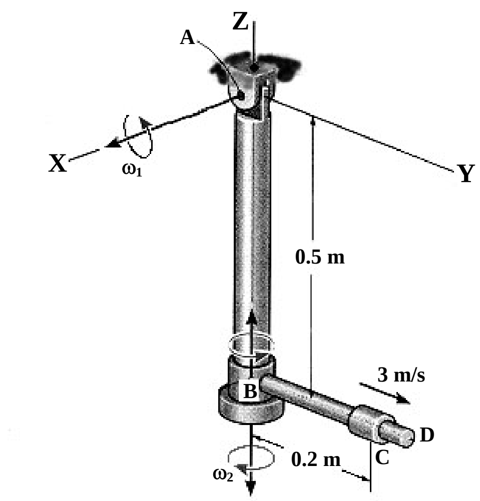


**Name** ______________________________ **Fall 2005**

## Qualifying Exam: Dynamics

# CLOSED BOOK

4. The pendulum shown below consists of two rods; ***AB*** is pin supported at **A** and swings only in the ***Y-Z plane***, whereas a bearing at **B** allows the attached rod ***BD*** to spin about rod ***AB***. At a given instant, the rods have the constant angular velocities $\boldsymbol{\omega_1 = 4}$ **rad/s** and $\boldsymbol{\omega_2 = 5}$ **rad/s**. Also, a collar **C**, located **0.2 m** from **B**, has a constant velocity of **3 m/s** with respect to the rod. Determine (a) the velocity of the collar and (b) the acceleration of the collar at the instant shown.



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**Qualifying Examination**
**Subject: Dynamics**

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

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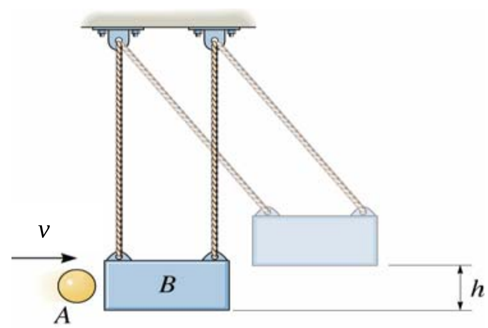
## Problem 1:

A ball with mass $m_A$ is thrown at a speed of $v$ toward a suspended block of mass $m_B$. Assume the block is at rest before it is hit by the ball; the coefficient of restitution between the ball and the block is $e$; and the gravitational acceleration is $g$. Determine:

(a) the maximum height $h$ to which the block will swing after the impact, and
(b) the minimum $e$ such that the ball will continue to move forward after the impact?

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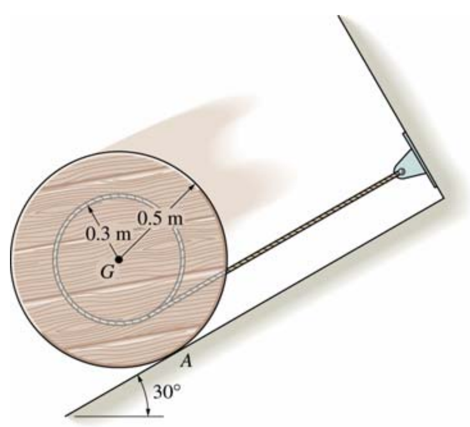
## Problem 2:

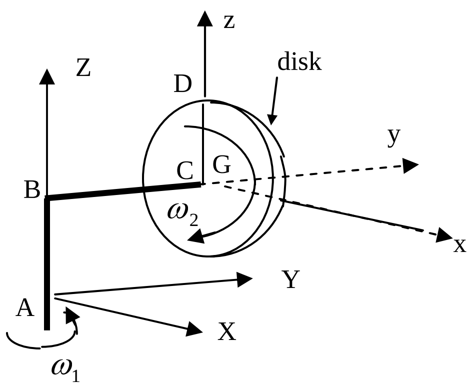
The spool has a mass of 60 kg and a radius of gyration of $K_G = 0.3$ m. Assume the kinetic friction coefficient between the spool and the inclined surface is $\mu = 0.2$. If the spool is released from rest, determine the angular velocity $\omega$ of the spool when its center $G$ moved $s_G = 1$ meter from its original position.

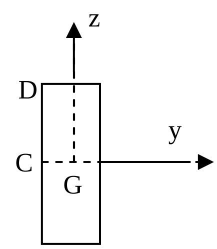


NAME: ____________________ Fall 2004

## Problem 3:

The disk of mass *m* and radius *r* spins about the rigid and massless shaft BC with an angular velocity of $\overline{\omega}_2$ and $\omega_2$ is constant. The support structure ABC rotates about the *Z*-axis at constant $\overline{\omega}_1$. The support structure ABC is also massless and rigid. The length of the shaft BC is $l_{CB}$. The coordinate system *XYZ* is fixed in space and the coordinate system *xyz* is 'welded' to the disk with its origins at the mass center G. At the instant shown the *z* and the *Z* axes are parallel, also the *y* and the *Y* axes are parallel. Determine the angular momentum $\overline{H}_G$ and the kinetic energy of the disk. The distance between points G and B is $l_{GB}$.




Side view of the disk

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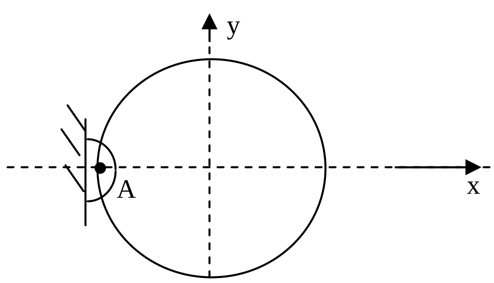
## Problem 4:

The disk has a mass of *m* and a radius of *r*. It is supported by a pin at point A. If the disk were released from rest while it was at the position shown in the diagram, determine

(a) the angular acceleration of the disk and
(b) the reaction at point A at the instant shown.

The magnitude of the acceleration due to gravity is *g* and the mass moment of inertia of the disk about its center of mass is $I_{ZG} = \frac{1}{2} mr^2$ .




Vertical plane

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**Qualifying Examination**
**Subject: Dynamics**

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

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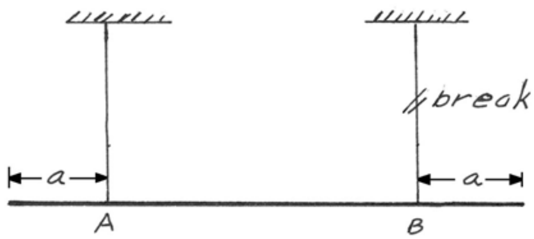
## Problem 1:

A uniform beam 10 feet long is held by two symmetrically spaced vertical cables. One of the cables breaks. Immediately after the break the tension in the intact cable is the same as it was before the break. How far from the end of the beam is the intact cable attached?

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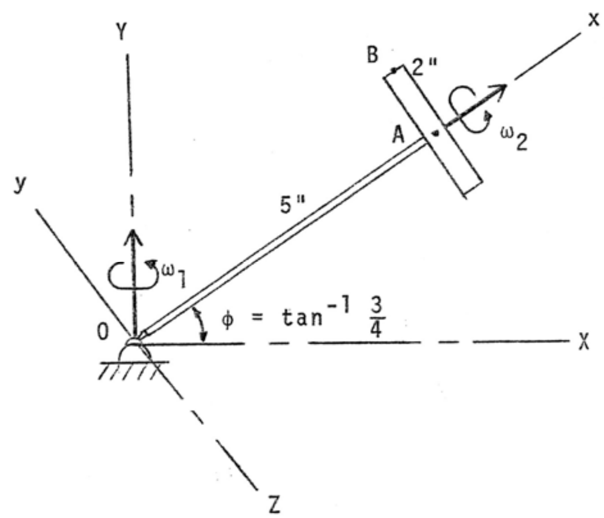
## Problem 2:

A gyroscope consists of a thin disk weighing 30 lb and of 2-in. radius rotating at a constant rate of $\omega_2 = 40$ rad/sec about axis 0A. Simultaneously, axis 0A is precessing (i.e., rotating) about a fixed vertical Y axis at the constant rate of $\omega_1 = 2$ rad/sec. Assuming the angle $\phi$ is constant, find

a) the angular velocity of the disk,
b) the angular acceleration of the disk,
c) the angular momentum of the gyroscope about the fixed point 0, and
d) the dynamic reaction at 0.

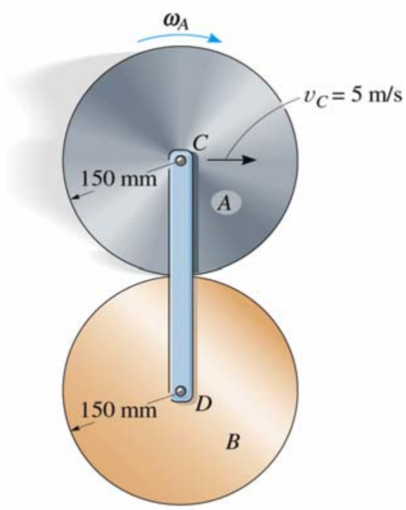


NAME: ______________________ Fall 2003

## Problem 3:

Disk *A* rolls without slipping over the surface of a fixed cylinder *B*. Both *A* and *B* have the same radius of 0.15 meters. If the center of *A* has a speed $v_c = 5$ m/s,

a) determine the angular velocities of disk *A* and link *CD*; and
b) how many revolutions will *A* have made about its center when link *CD* just completes one revolution? [Hint: rotation of *A* about its own center *C* is equivalent to the relative rotation of *A* with respect to link *CD* in this case]

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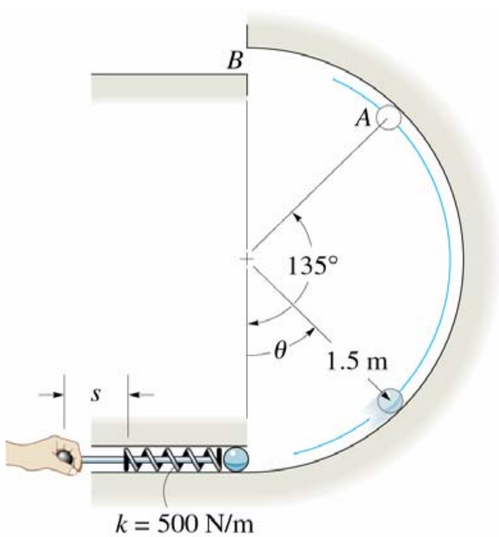
## Problem 4:

The 0.5-kg ball of negligible size is fired up the vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08m when $s = 0$. How far *s* must the plunger be pulled back and released so that the ball will just make it around and land on the platform at *B*?



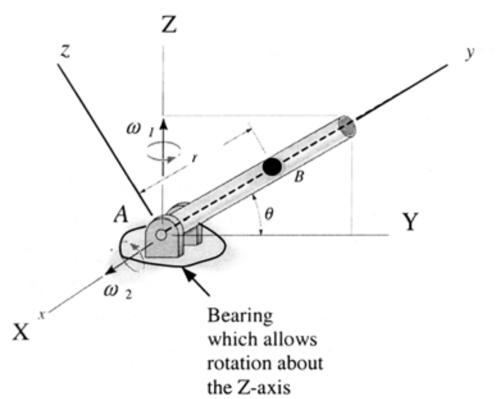
# Qualifying Exam Spring 2003
# Dynamics

This portion of the qualifying exam is **closed** book. You may have a calculator.

**There are 4 problems here. Pick any 3 to work.**

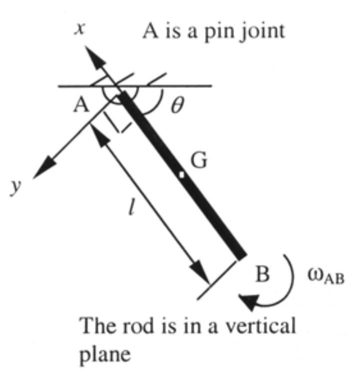
## Problem 1:

The tube is rotating about the Z-axis with a constant angular velocity of $\omega_1$, while at the same instant the tube is rotating upward about the x-axis at a constant rate $\omega_2$. If the ball B is blown through the tube at a constant rate of $\dot{r}$, determine the velocity and acceleration of the ball B at the instant shown. Neglect the size of the ball. Coordinate system xyz is welded to the tube and coordinate system XYZ is fixed in space. At the instant shown, axes y, z, Y, and Z are on the same plane. You must use only the coordinate systems given.



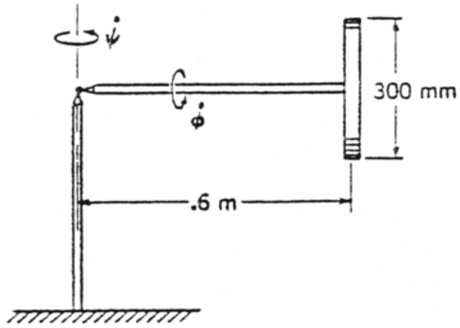

## Problem 2:

The uniform slender rod AB has a mass m and length $l$. When it is at the position shown its angular speed is $\omega_{AB}$. At the instant shown, determine (a) the acceleration of the mass center G and (b) the x and y components of the force pin A exerts on the rod. Neglect the friction at the pin joint.



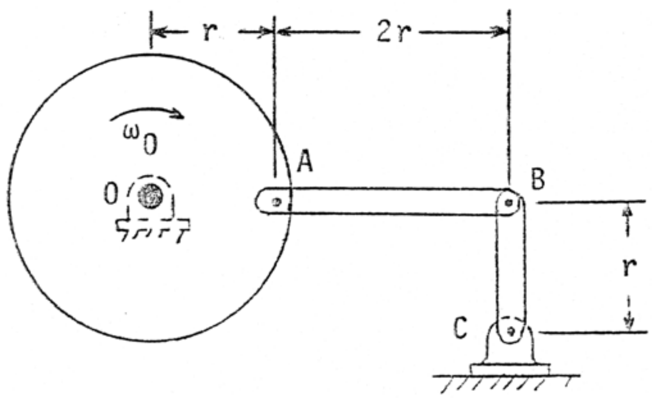
## Problem 3:

The disc shown is undergoing regular precession as shown at the rate of $\dot{\psi} = 0.3$ rad/s. Determine the spin velocity $\dot{\phi}$ required to sustain this motion. The disc weighs 90 N, and the mass of the rod is negligible.




## Problem 4:

At the instant shown, the disk rotates with a constant angular velocity $\omega_o$ clockwise. Determine the angular velocities and the angular accelerations of the rods AB and BC.



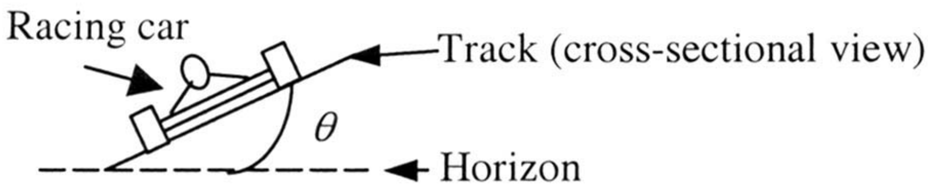
# Qualifying Exam Fall 2002
# Dynamics

This portion of the qualifying exam is **closed** book. You may have a calculator.

**There are 4 problems here. Pick any 3 to work.**

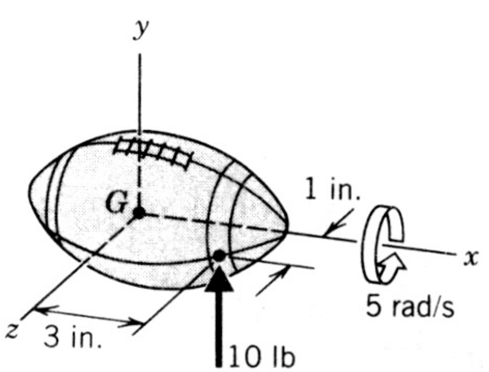
## Problem 1:

A racing car of mass $m$ travels around a circular track of radius $\rho$ with a speed $v$. Draw a free body diagram of the car. Determine the banking angle $\theta$ for the race track such that the racing car has no tendency to slide up or down the track. Assume that the racing car is a particle.




## Problem 2:

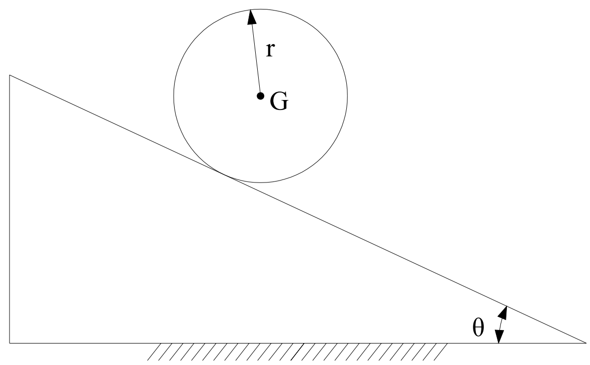
A 2-lb football is in free flight, traveling horizontally at 60 ft/s as it spins about its x axis at 5 rad/s. A would be receiver deflects the ball, applying a 10-lb force as shown for an interval of 0.10s. Determine the linear and angular velocity of the football after it is deflected. The radii of gyration are $k_x$ = 2in, $k_y$ = $k_z$ = 3in.



## Problem 3:

Determine the minimum value of the coefficient of friction required between the cylinder and the incline plane for the cylinder to roll without slipping.

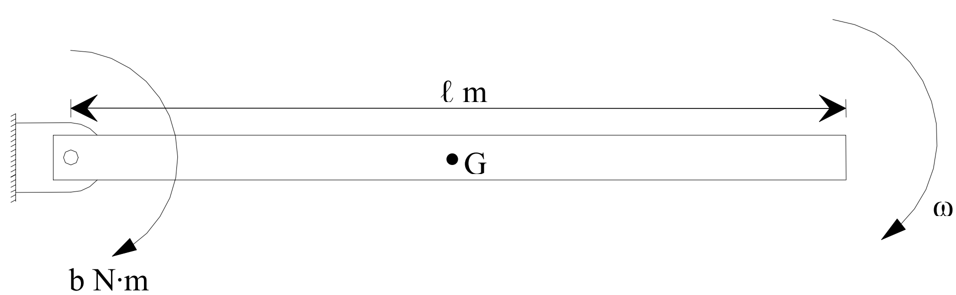
$I_G = (1/2)mr^2$




## Problem 4:

The slender rod has a mass of *m* kg and a length of *ℓ* m. It is subjected to a torque of magnitude *b* N·m as shown and is rotating in the vertical plane. At the instant shown it has an angular speed of *ω* rad/s and the direction of rotation is as shown in the diagram. Draw a free body diagram of the rod. Also determine the angular acceleration of the rod and the reaction exerted by the pin O on the rod.

$I_G = (1/12)m\ell^2$

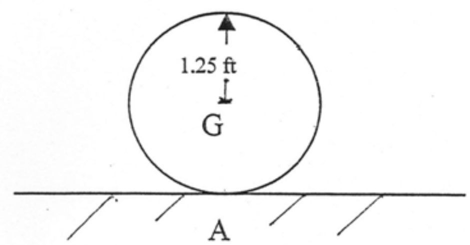


# Qualifying Exam Spring 1999
# Dynamics

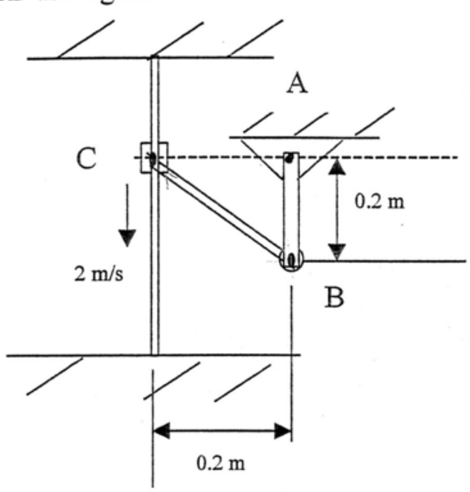
This portion of the qualifying exam is **closed** book.  You may have a calculator.

**There are 4 problems here.  Pick any 3 to work.**

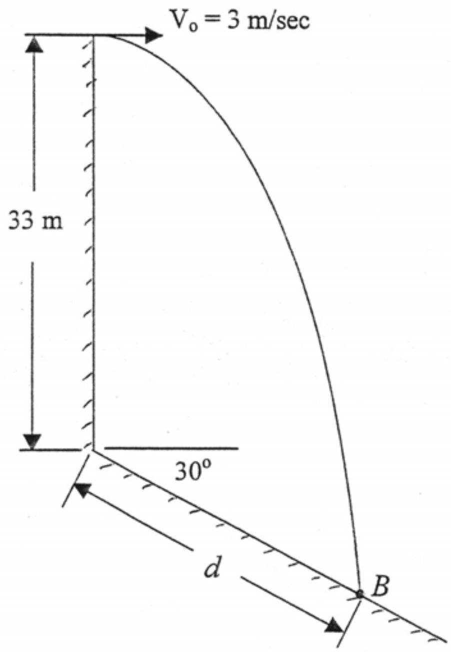
(1) The 50-lb wheel has a radius of gyration $k_G$=*0.70 ft*. If a 35 ft-lb couple moment is applied to the wheel, determine the acceleration of its mass center G. The coefficients of static and kinetic friction between the wheel and the plane at A are $\mu_s$=*0.30* and $\mu_k$=*0.25* respectively.



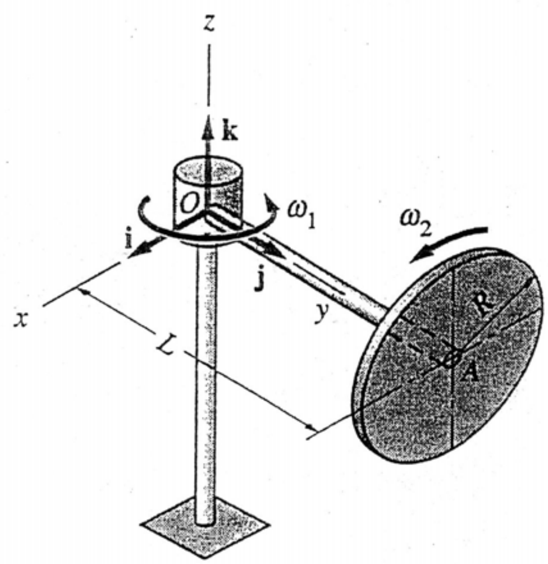

(2) The collar C is moving downward with a speed of *2 m/s* and increasing at a rate of *1 m/s*$^2$. At the instant shown, determine (1) the angular velocities of AB and CB and (2) the angular accelerations of AB and CB. Arms AB and CB are rigid.



Problem 3. Stones are thrown off the cliff with a horizontal velocity of 3m/sec. Determine the distance $d$ down the slope to where the stones hit the ground at $B$.




Problem 4. The arm $OA$ of the system shown is rotating about the vertical shaft with the angular velocity $\omega_1$. At the same time, the thin disk at $A$ is spinning with angular velocity $\omega_2$ relative to $OA$. Determine the angular velocity $\omega$ and the angular acceleration $\alpha$ of the disk in the position shown. Assume $\omega_1$ and $\omega_2$ are not constants.



# Qualifying Exam Fall 1998
# Dynamics

This portion of the qualifying exam is **closed** book. You may have a calculator.

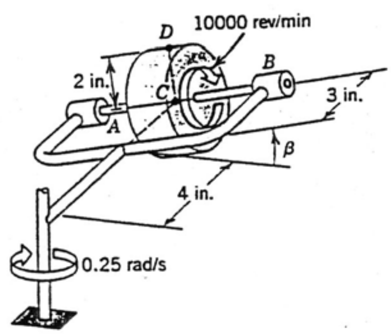
**There are 4 problems here. Pick any 3 to work.**

**Problem 1:**

The gyroscopic turn indicator consists of a flywheel that spins about axis AB at 10,000 rev/min, as the assembly rotates about the vertical axis at the constant rate of 0.25 rad/s. The angle of tilt is constant at $\beta = 20°$. Determine:

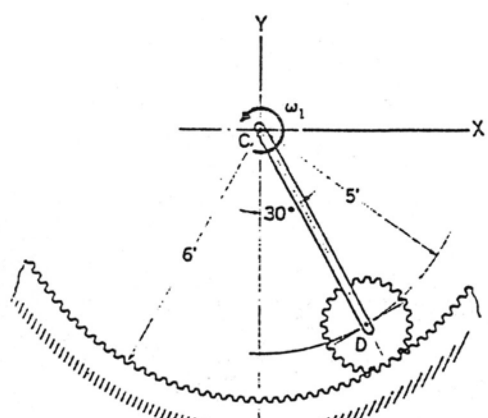
a.) the angular velocity and angular acceleration of the disk, and

b.) the velocity and acceleration of point D on the flywheel.



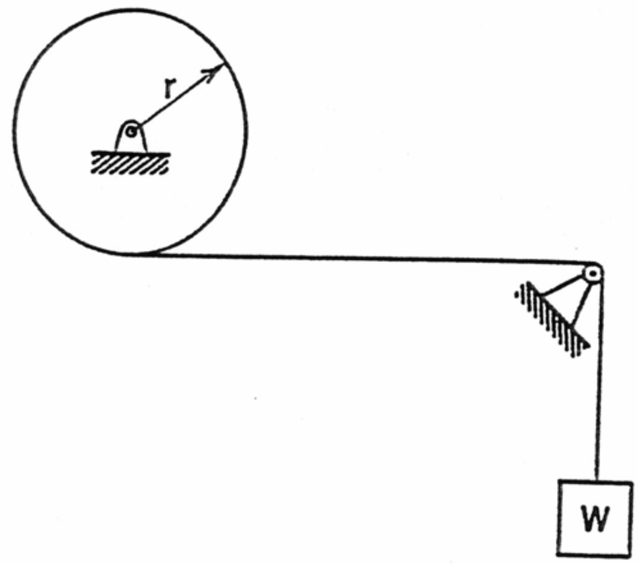

**Problem 2:**

If $\omega_1 = 5$ rad/s and $\dot{\omega}_1 = 3$ rad/s$^2$ for bar CD, compute the angular velocity and angular acceleration of the gear D.



**Problem 3:**

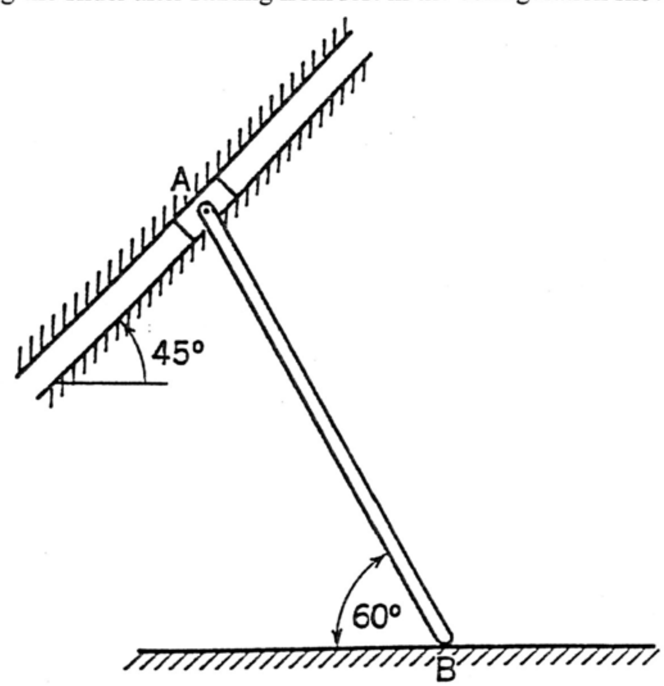
Shown is a cylinder connected to a weight by an inextensible cord that runs over a tiny pulley, whose mass can be neglected. As the weight falls, it causes the cylinder to rotate about its axis. When the weight is released from rest, what is the angular acceleration of the cylinder and the linear acceleration of the weight?




**Problem 4:**

A 10 ft. rod AB weighing 50 lb is guided at A by a slot and at B by a horizontal surface. Neglecting the mass of the slider at A, what is the speed of B when A has moved 3 ft. along the slider after starting from rest in the configuration shown.

friction? say negl.

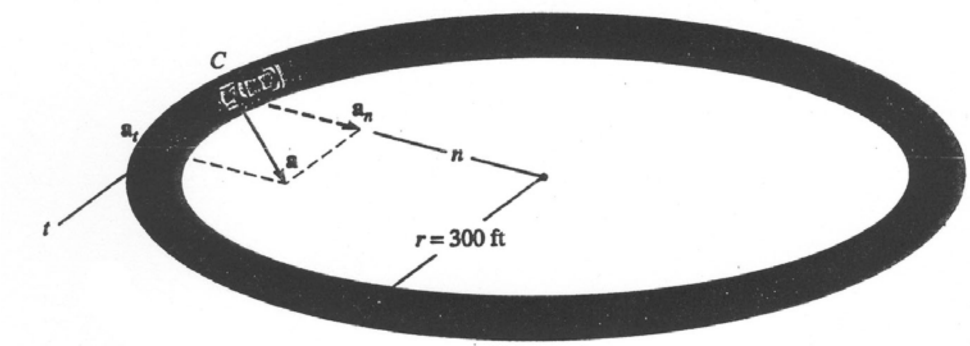


# Qualifying Exam Spring 1997
# Dynamics

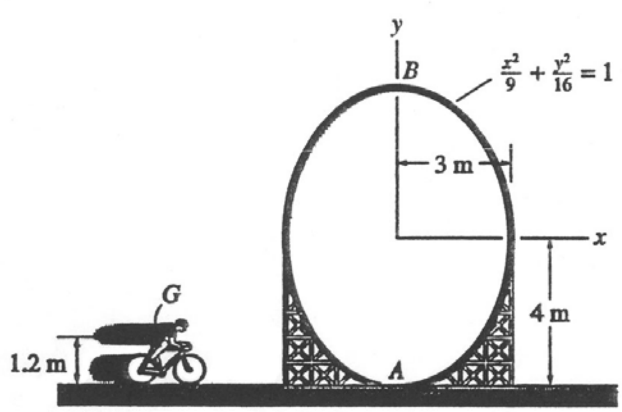
This portion of the qualifying exam is **closed** book. You may have a calculator.

**There are 4 problems here. Pick any 3 to work.**

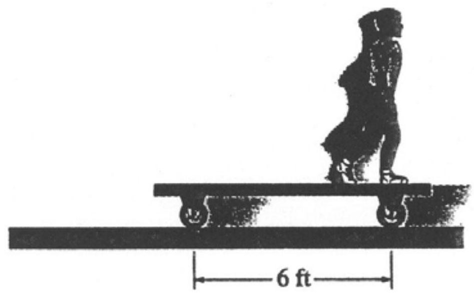
1. A race car $C$ travels around the horizontal circular track that has a radius of 300 ft. If the car increases its speed at a constant rate of 7 ft/s$^2$, starting from rest, determine the time needed for it to reach an acceleration of 8 ft/s$^2$. What is its speed at this instant?



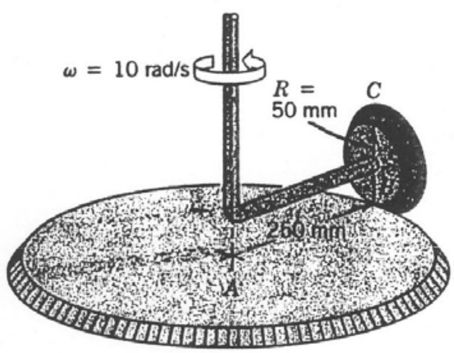

2. The man on the bicycle attempts to coast around the ellipsoidal loop without falling off the track. Determine the speed he must attain at $A$ just before entering the loop in order to perform the stunt. The bicycle and man have a total mass of 85 kg and a center of mass at $G$. Neglect the mass of the wheels.



3. A 100-lb boy walks forward over the surface of the 60-lb cart with a constant speed of 3 ft/s relative to the cart. Determine the cart's speed and its displacement at the moment he is about to step off. Neglect the mass of the wheels.




4. Gear $B$ rotates freely about its shaft, which has a constant angular speed of 10 rad/s about the vertical axis. Gear $A$ is stationary. Determine (a) the angular velocity and angular acceleration of gear $B$, and (b) the velocity and acceleration of point $C$.

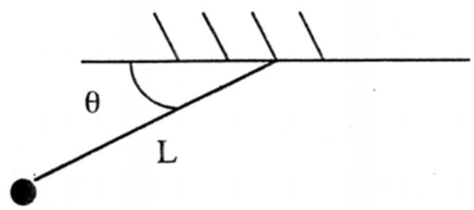


# Qualifying Exam Spring 1996
# Dynamics

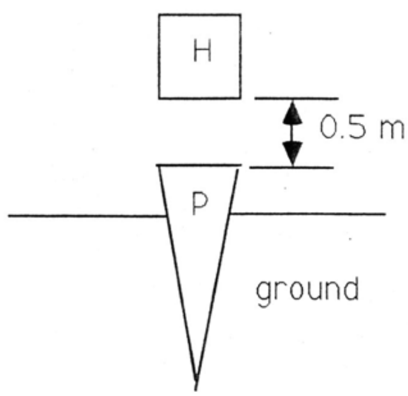
This portion of the qualifying exam is **closed** book. You may have a calculator.

**There are 3 problems here. Complete all problems**

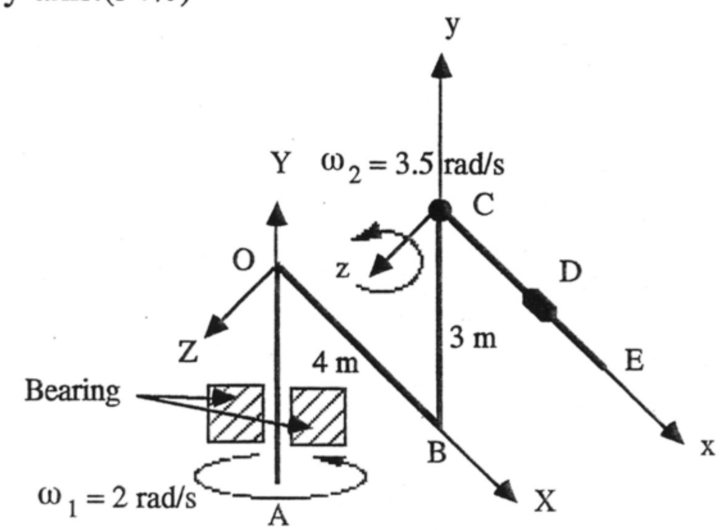
[1] A pendulum bob of mass m is released from rest when $\theta = 0^{o}$, with the cord taut. Determine the magnitude of the tension in the cord as a function of $\theta$. Neglect the size of the bob. (33%)




[2] A rigid pile P with a mass of 800 kg is driven into the ground using a hammer H. The mass of the hammer is 300 kg. It is 0.5 m above the top of the pile when it is released from rest and strikes the top of the pile with no rebound off the pile. Determine the impulse which the hammer imparts on the pile. (33%)



[3] Collar D slides without friction on the smooth rod CE. At a distance of 2 m from point C, collar D has a speed of 3 m/s and increasing at a rate of 0.5 m/s$^2$ relative to rod CE. Rod CE rotates about point C in the xy-plane with a constant angular speed of $\omega_2$ = 3.5 rad/s in the counter-clockwise direction. At the same time structure AOBC rotates about the Y-axis counter-clockwise with a constant speed of $\omega_1$ = 2 rad/s. Coordinate system XYZ is fixed in space and coordinate system xyz is welded to rod CE. Based on these coordinate systems, determine (a) the velocity of collar D, (b) the angular acceleration of rod CE and (c) the acceleration of collar D. The lengths of OB and BC are 4 m and 3 m respectively. At the instant shown, X-axis is parallel to x-axis and Y-axis is parallel to y-axis.(34%)

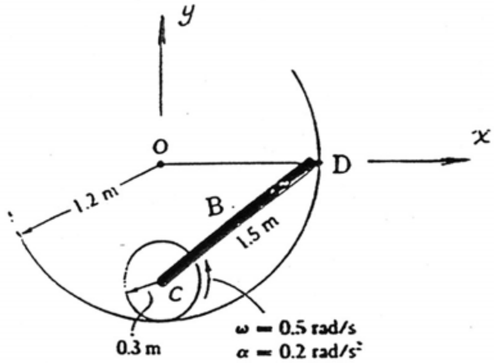


# Qualifying Exam Spring 1995
# Dynamics

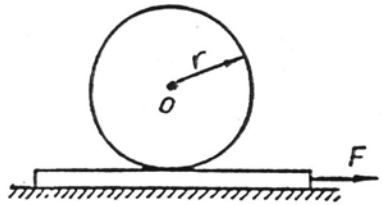
This portion of the qualifying exam is **closed** book. You may have a calculator.

**There are 4 problems here. Complete problems 1 and 2 and either 3 or 4.**

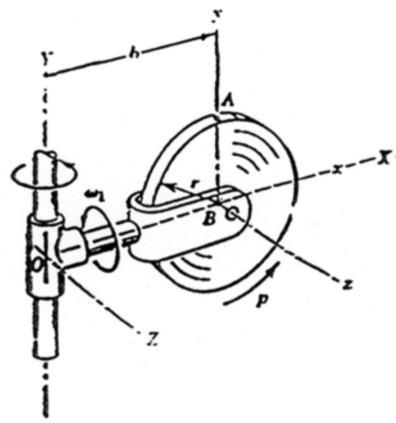
**Problem #1** The cylinder A is rolling on the fixed circular track with given angular velocity and acceleration. Rod B is pinned to the center C of cylinder and its other end D slides on the track. Find the velocity and acceleration of D when the cylinder is at the bottom of the track.



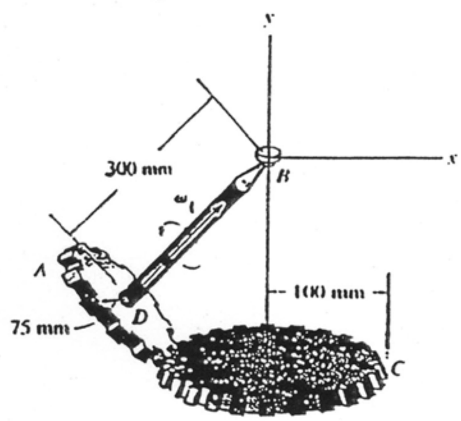

**Problem#2** The force $\bar{F}$ is acting on the plate shown in diagram. At the same time the uniform disk is rolling on the plate without slipping. The kinetic coefficient of friction between plate and ground is $\mu_k$. Given the masses of plate and disk are $m_1$ and $m_2$, respectively. Determine accelerations of the plate and the center point of disk.



Problem #3 The thin disk of radius r is rotating about its z-axis with a constant angular elocity p, and yoke in which it is mounted rotates about x-axis through OB with a constant ngular velocity $\omega_1$. Simutaneously, the entire assembly rotates about Y-axis with constant ngular velocity $\omega_2$. Determine the velocity and acceleratin of point A.




Problem #4 The shaft BD is connected to a ball-and-socket joint at B, and a beveled gear A is attached to its other end. The gear is in mesh with a fixed gear C. If the shaft and gear A are spinning with a constant angular velocity of $\omega_1 = 8$ rad/s, determine the angular velocity and angular acceleration of gear A.



# Qualifying Exam Spring 1993
# Dynamics

This portion of the qualifying exam is **closed** book. You may have a calculator.

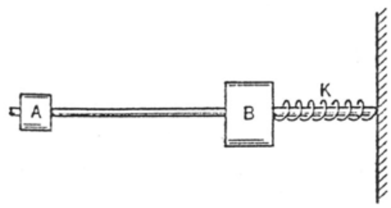
**There are 4 problems here. Pick any 3 to work.**

## Question #1

Cylinder A, weighing 10 lb, moves toward cylinder B, weighing 40 lb, at the speed of 20 ft/s . Mass B is attached to a spring having a spring constant K equal to 10 lb/in. The collision has a coefficient of restitution $\varepsilon = 0.9$. Assume that there is no friction along the rod and that the spring has negligible mass. Determine:

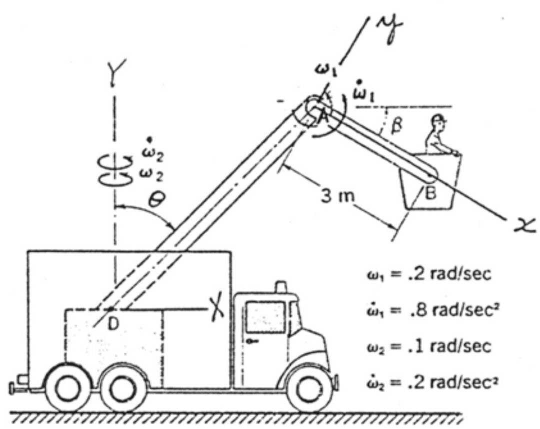
(a) The velocity of cylinder B after impact,

(b) The maximum deflection of the spring.



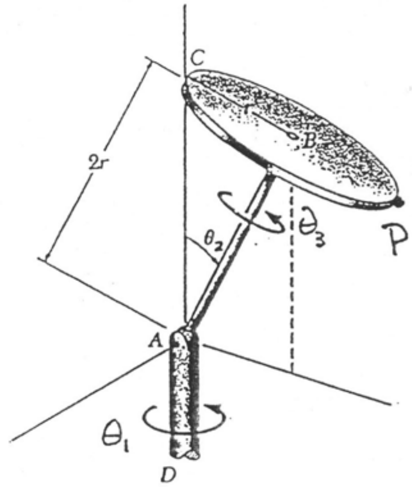

## Question #2

A stationary truck is carrying a cockpit for a worker who repairs overhead fixtures. At the instant shown, the base D is rotating at angular speed $\omega_2 = 0.1$ rad/s with $\dot{\omega}_2 = 0.2$ rad/s$^2$ relative to the truck. Arm AB is rotating at angular speed $\omega_1$ of 0.2 rad/s with $\dot{\omega}_1 = 0.8$ rad/s$^2$ relative to DA. Determine $\bar{\omega}_{AB}$, $\bar{\alpha}_{AB}$, $\bar{v}_B$ when $\theta = 45°$ and $\beta = 30°$
Take DA = 13 m.



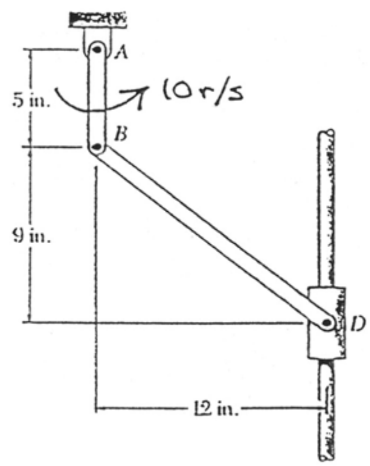
## Question #3

Referring to the figure, find the velocity of the point P in terms of r, $\theta_1$, $\dot{\theta}_1$, $\theta_2$, $\dot{\theta}_2$, $\theta_3$, $\dot{\theta}_3$.




## Question #4

Referring to the figure, find the velocity of the point D at the instant shown.

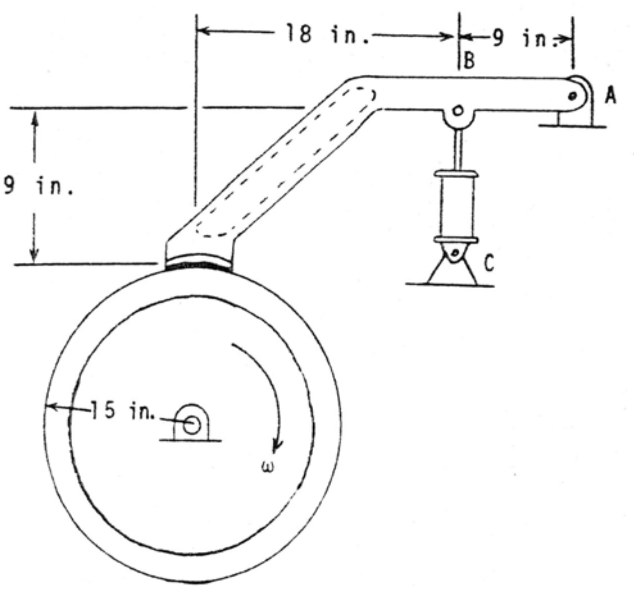


# Qualifying Exam
# Dynamics

Additional Problems

**Question 2.**

The 15 inch radius brake drum is attached to a larger flywheel which is not shown. The total mass moment of inertia of the flywheel and drum is $15lb - ft - s^2$. Knowing that the initial angular velocity is 150 rpm clockwise, determine the force which must be exerted by the hydraulic cylinder BC is the system is to stop in 10 revolutions. The coefficient of friction between the arm and the drum is $\mu = 0.4$.




**Question 3.**

Determine the required tension $T$ is the accerlation of the $500lb$ block $A$ is to be $6ft/s^2$ upward. The pulleys are of negligible mass.