Qualifying Exam, Fall 2022
Solid Mechanics

* This is a closed-book test (with a cheat sheet provided), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #_____, #_____, and #_____ to be graded.

Given:

A cantilever beam with a C-channel cross section is loaded by a shear force P, an axial force F and a torque T as shown below.

a. Identify location along the length of the beam with the highest magnitude of normal stresses in the given coordinate system
b. For that location, find all components of stress at points A (top point) and B (neutral axis) of the cross section
c. For points A and B, calculate Von-Mises stresses and safety factors based on the given yield strength

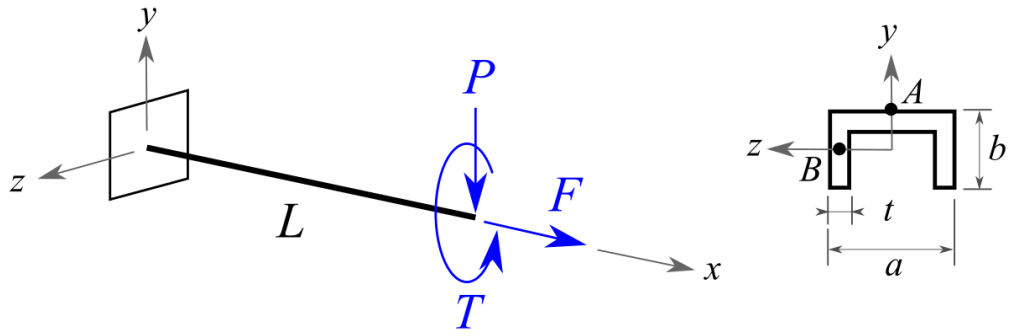
Assume:

$$L = 20\ cm, a = 50\ mm, b = 30\ mm, t = 3\ mm, P = 0.4\ kN, F = 20\ kN, T = 20\ N \cdot m$$

$\bar{y} = 9.3\ mm$ (distance from the top to the neutral axis of the cross section)

$$I_z = 1.229 \cdot 10^4\ mm^4, S_y = 200\ MPa$$

Do not neglect the shear stress due to the shear force



Relevant formulas:

- Open thin-walled sections:

$$\tau = G\theta_1 c = \frac{3T}{Lc^2}$$

- Von-Mises stress (general xyz components):

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]^{1/2}$$

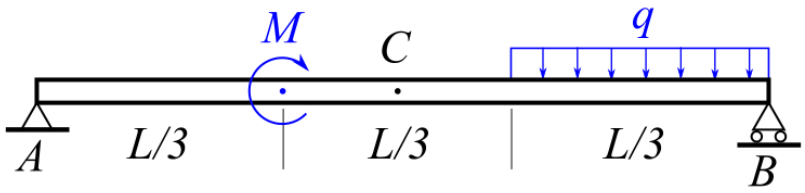
Given:

A simply supported beam shown below.

a) Use Castigliano's theorem to find deflection at point C.
b) Write down the integral required for finding the slope at C, no need to evaluate it

Assume:

$$L = 1.2\ m, I = 12 \cdot 10^{-7}\ m^4, E = 70\ GPa, M = 7\ kN \cdot m, q = 3\ kN/m$$



Relevant formulas:

$$\delta_i = \frac{\partial U}{\partial F_i} = \int \frac{1}{EI}\left(M\frac{\partial M}{\partial F_i}\right)dx$$

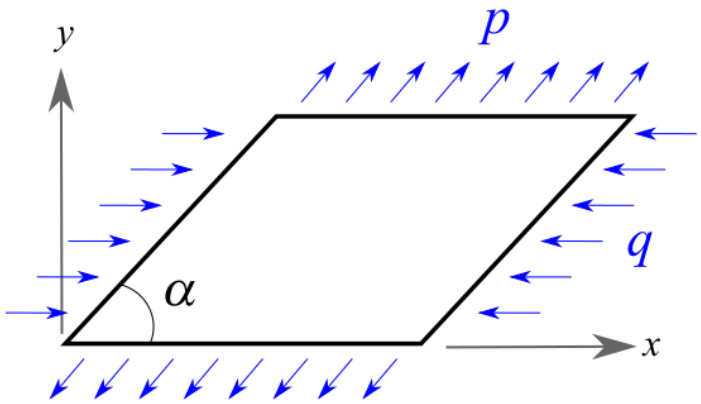
Given:

A think skewed plate is subjected to uniform distribution of stress along its sides as shown below.

a) Calculate stress components σ_x, σ_y and τ_{xy} in the plate
b) Find principal stresses and their orientations
c) Plot 3D Mohr's circle
d) Determine the absolute max shear stress

Assume:

$$p = 30\ MPa, q = 18\ MPa, \alpha = 60°$$



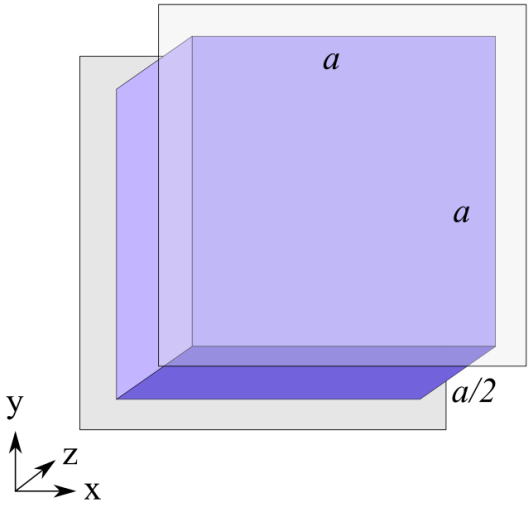
Given:

A box fixed between two smooth rigid plates as shown below. The plates prevent any motion in the z-direction, but do not affect motion in x or y direction. Initially the box is stress-free, a temperature change of ΔT is applied to the box.

a) Find all components of stress and strain
b) Calculate the normal force exerted by the box on either one of the plates
c) What are the principal stresses in the box?
d) What are the principal strains in the box?
e) Calculate max shear strains in the xy plane? Absolute max shear strains in the box?

Assume:

$$E = 115\ GPa, \nu = 0.3, \alpha(CTE) = 8 \cdot 10^{-6}\frac{1}{°C}, a = 1.6\ cm, \Delta T = 300°C$$



Relevant formulas:

$$\varepsilon_{ij} = \frac{1}{E}\left[(1+\nu)\sigma_{ij} - \nu\sigma_{kk}\delta_{ij}\right] + \alpha\Delta T\delta_{ij}$$