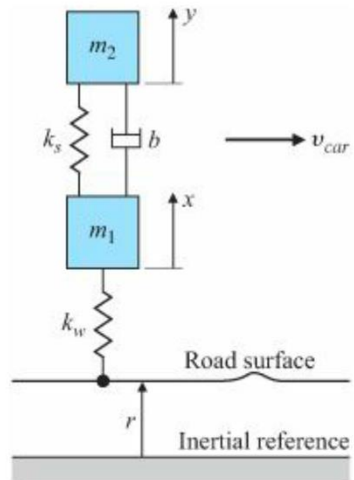
Qualifying Exam, Fall 2022
Control

* This is a closed-book test (with a cheat sheet provided), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #____________, #___________, and #____________ to be graded.

Problem 1. Consider the automobile suspension system shown in the right figure. Assume the masses, $m_1 = m_2 = 1$ kg, the damper constant, $b = 1$ Ns/m, and the spring constants $k_s = k_w = 1$ N/m. Define the system input as $r(t)$ and the system output as $y(t)$.



(1) Find the equations of motion for the system. (5 pts)

(2) Find the transfer function $\frac{Y(s)}{R(s)}$. (5 pts)

Problem 2. Consider the linear system $\dot{x} = Ax + Bu,\ y = Cx$, where

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0]$$

(1) Is the system controllable? Why or Why not? (5 pts)

(2) Is the system observable? Why or why not? (5 pts)

Problem 3. For a rigid spacecraft with principal moments of inertia (kgm^2):

$$J_x = 3000,\ J_y = 2000,\ J_z = 1000,$$

and the equations of the rotational motion of the spacecraft without external torque can be obtained as

$$\dot{\omega}_x = \left(\frac{J_y - J_z}{J_x}\right)\omega_y\omega_z, \qquad \dot{\omega}_y = \left(\frac{J_z - J_x}{J_y}\right)\omega_z\omega_x, \qquad \dot{\omega}_z = \left(\frac{J_x - J_y}{J_z}\right)\omega_x\omega_y,$$

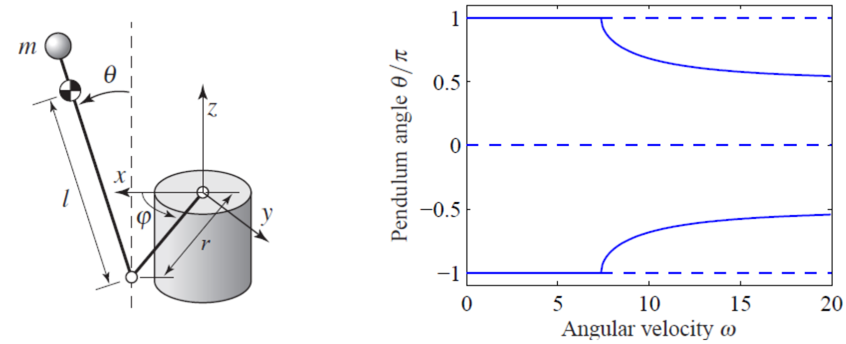
Let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$ and an equilibrium point is $x_e = \begin{bmatrix} x_{e1} \\ x_{e2} \\ x_{e3} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ (rad/s).

(1) Derive linearized equations and state-space model of $\dot{x} = Ax$. (5 pts)
(2) Analyze the stability of the linear system $\dot{x} = Ax$. (5 pts)

Problem 4. The Furuta pendulum, an inverted pendulum on a rotating arm, is shown to the left in the figure below. The equations of motion for the system are given by

$$J\ddot{\theta} - J\omega^2 \sin\theta\cos\theta - mgl\sin\theta = 0,$$

where J is the moment of inertia of the pendulum with respect to its pivot, m is the pendulum mass, l is the distance between the pivot and the center of mass of the pendulum, and ω is the rate of rotation of the arm



(1) Derive the state equations using $x_1$and x_2. (5 pts)

(2) Determine the equilibria for the Furuta pendulum system where $J = 1$ $[kgm^2]$, $\omega = 3$ $[rad/s]$, $m = 4$ $[kg]$, $g = 9.8$ $[m/s^2]$, and $l = 0.3$ $[m]$. (5 pts)