Fall 2019 Qualifying Exam - Conduction Heat Transfer (Q1) - Closed Book
Transient Temperature Distribution in a Thin Slab
A very large (in $Y$ and $Z$ directions) thin slab is used inside the wall of a building. It is made of a porous material and has a thickness of 2 cm (in the $X$ direction). It is initially at $40^{\circ} \mathrm{C}$ on a hot day. Suddenly, the temperatures of both the large surfaces are reduced to $30^{\circ} \mathrm{C}$ for all $t>0$ where $t$ is the time in seconds. Develop an analytical expression for the transient temperature distribution $T(x, t)$ in the porous slab. Assume the effective density of the slab is $850 \mathrm{~kg} / \mathrm{m}^{3}$ and the effective specific heat is $1900 \mathrm{~J} / \mathrm{kg} \cdot \mathrm{K}$.

Equations

$$
\begin{aligned}
& \frac{\partial}{\partial \mathrm{x}}\left(k \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right)+q=\rho \cdot c_{p} \frac{\partial T}{\partial t} \\
& \frac{\partial^{2} x}{\partial x^{2}}+\lambda^{2} x=0 ; \text { solution: } X(x)= A \cos (\lambda x) \\
&+B \sin (\lambda x) \\
& \frac{\partial^{2} x}{\partial x^{2}}-\lambda^{2} x=0 ; \text { solution: } x(x)=A e^{-\lambda x}+B e^{\lambda x} \\
& \frac{\partial x}{\partial t}+c \lambda^{2} x=0 ; \text { solution: } x(t)=D e^{-c \lambda^{2} t} \\
& \sin ^{2}(x)= \frac{1-\cos (2 x)}{2}
\end{aligned}
$$

## Fall 2019 Qualifying Exam - Radiation Heat Transfer (Q2) - Closed Book

## Real Body Radiation

Two parallel plates 0.5 m X 1 m are spaced 0.5 m apart. One plate is maintained at $1000^{\circ} \mathrm{C}$ and the other at $500^{\circ} \mathrm{C}$. The plates have emissivities of 0.2 and 0.5 respectively and are located in a very large room whose walls are at $27^{\circ} \mathrm{C}$. The surfaces of the plates that are facing each other (surfaces 1 and 2 ) are exchanging heat between them and the room (surface 3 ). The backsides of the plates are thoroughly insulated. Find the net heat transfer rate to each plate and to the room.



## Fall 2019 Qualifying Exam - Thermodynamics (Q3) - Closed Book

## Gas Mixtures

(a) A mixture of $60 \% \mathrm{~N}_{2}, 30 \% \mathrm{Ar}$ and $10 \% \mathrm{O}_{2}$ on a mass basis is in a cylinder at $250 \mathrm{kPa}, 310 \mathrm{~K}$ and volume $0.5 \mathrm{~m}^{3}$. Find the mole and the mass fractions and the mass of argon.
(b) A slightly oxygenated air mixture is $69 \% \mathrm{~N}_{2}, 1 \% \mathrm{Ar}$ and $30 \% \mathrm{O}_{2}$ on a mole basis. Assume a total pressure of 101 kPa and find the mass fraction of oxygen and its partial pressure.

$$
\begin{array}{rr}
\mathrm{mf}_{i}=\frac{m_{i}}{m_{m}} \text { and } y_{i}=\frac{N_{i}}{N_{m}} & M_{m}=\frac{m_{m}}{N_{m}}=\sum_{i=1}^{k} y_{i} M_{i} \text { and } R_{m}=\frac{R_{u}}{M_{m}} \\
m_{m}=\sum_{i=1}^{k} m_{i} \text { and } N_{m}=\sum_{i=1}^{k} N_{i} & \mathrm{mf}_{i}=y_{i} \frac{M_{i}}{M_{m}} \text { and } M_{m}=\frac{1}{\sum_{i=1}^{k} \frac{\mathrm{mf}_{i}}{M_{i}}} \\
\frac{P_{i}}{P_{m}}=\frac{V_{i}}{V_{m}}=\frac{N_{i}}{N_{m}}=y_{i} & \mathrm{R}_{\mathrm{m}}=\Sigma \mathrm{c}_{\mathrm{i}} \cdot \mathrm{R}_{\mathrm{i}} \quad P V=R T \quad R_{u}=8.31447 \mathrm{~kJ} / \mathrm{kmol} \cdot \mathrm{~K}
\end{array}
$$

| Molar mass, gas constant, and critical-point properties |  |  |  |
| :---: | :---: | :---: | :---: |
| Substance | Formula | Molar mass, $M \mathrm{~kg} / \mathrm{kmol}$ | Gas constant, $R \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}^{*}$ |
| Air | - | 28.97 | 0.2870 |
| Ammonia | $\mathrm{NH}_{3}$ | 17.03 | 0.4882 |
| Argon | Ar | 39.948 | 0.2081 |
| Benzene | $\mathrm{C}_{6} \mathrm{H}_{6}$ | 78.115 | 0.1064 |
| Bromine | $\mathrm{Br}_{2}$ | 159.808 | 0.0520 |
| $n$-Butane | $\mathrm{C}_{4} \mathrm{H}_{10}$ | 58.124 | 0.1430 |
| Carbon dioxide | $\mathrm{CO}_{2}$ | 44.01 | 0.1889 |
| Carbon monoxide | CO | 28.011 | 0.2968 |
| Carbon tetrachloride | $\mathrm{CCl}_{4}$ | 153.82 | 0.05405 |
| Chlorine | $\mathrm{Cl}_{2}$ | 70.906 | 0.1173 |
| Chloroform | $\mathrm{CHCl}_{3}$ | 119.38 | 0.06964 |
| Dichlorodifluoromethane (R-12) | $\mathrm{CCl}_{2} \mathrm{~F}_{2}$ | 120.91 | 0.06876 |
| Dichlorofluoromethane (R-21) | $\mathrm{CHCl}_{2} \mathrm{~F}$ | 102.92 | 0.08078 |
| Ethane | $\mathrm{C}_{2} \mathrm{H}_{6}$ | 30.070 | 0.2765 |
| Ethyl alcohol | $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}$ | 46.07 | 0.1805 |
| Ethylene | $\mathrm{C}_{2} \mathrm{H}_{4}$ | 28.054 | 0.2964 |
| Helium | He | 4.003 | 2.0769 |
| $n$-Hexane | $\mathrm{C}_{6} \mathrm{H}_{14}$ | 86.179 | 0.09647 |
| Hydrogen (normal) | $\mathrm{H}_{2}$ | 2.016 | 4.1240 |
| Krypton | Kr | 83.80 | 0.09921 |
| Methane | $\mathrm{CH}_{4}$ | 16.043 | 0.5182 |
| Methyl alcohol | $\mathrm{CH}_{3} \mathrm{OH}$ | 32.042 | 0.2595 |
| Methyl chloride | $\mathrm{CH}_{3} \mathrm{Cl}$ | 50.488 | 0.1647 |
| Neon | Ne | 20.183 | 0.4119 |
| Nitrogen | $\mathrm{N}_{2}$ | 28.013 | 0.2968 |
| Nitrous oxide | $\mathrm{N}_{2} \mathrm{O}$ | 44.013 | 0.1889 |
| Oxygen | $\mathrm{O}_{2}$ | 31.999 | 0.2598 |
| Propane | $\mathrm{C}_{3} \mathrm{H}_{8}$ | 44.097 | 0.1885 |
| Propylene | $\mathrm{C}_{3} \mathrm{H}_{6}$ | 42.081 | 0.1976 |
| Sulfur dioxide | $\mathrm{SO}_{2}$ | 64.063 | 0.1298 |
| Tetrafluoroethane (R-134a) | $\mathrm{CF}_{3} \mathrm{CH}_{2} \mathrm{~F}$ | 102.03 | 0.08149 |
| Trichlorofluoromethane (R-11) | $\mathrm{CCl}_{3} \mathrm{~F}$ | 137.37 | 0.06052 |
| Water | $\mathrm{H}_{2} \mathrm{O}$ | 18.015 | 0.4615 |
| Xenon | Xe | 131.30 | 0.06332 |

## Fall 2019 Qualifying Exam - Convection Heat Transfer (Q4) - Closed Book

## Laminar Flow through a Pipe

"Slug" flow in a pipe may be described as that flow in which the velocity is constant across the entire flow area of the tube, i.e., $u=u_{0}$ (constant). Obtain an expression for the heat-transfer coefficient and the Nusselt number in this type of flow with a constant-heat-flux condition maintained at the wall.

## Equations

$\alpha \cdot \frac{1}{r} \cdot \frac{\partial}{\partial \gamma}\left(\gamma \cdot \frac{\partial T}{\partial \gamma}\right)=u \cdot \frac{\partial T}{\partial z}$

$$
\begin{aligned}
& T_{m}: \text { Bulk Mean Temperature } \\
& \text { or } \\
& \text { Mixing } c_{u p} \text { Temperature } \\
& T_{m}= \frac{\oiint \rho \cdot c_{p} \cdot u \cdot T \cdot d A}{\oiint \rho \cdot c_{p} \cdot u \cdot d A} \\
& h= \frac{-k \cdot(\partial T / \partial r)_{r=R}}{T_{w}-T_{m}} \\
& \overline{N u}_{D}= \frac{\bar{h} \cdot D}{k}
\end{aligned}
$$

