Fall 2019 Qualifying Exam – Conduction Heat Transfer (Q1) – Closed Book

Transient Temperature Distribution in a Thin Slab

A very large (in Y and Z directions) thin slab is used inside the wall of a building. It is made of a porous material and has a thickness of 2 cm (in the X direction). It is initially at 40 $^{\circ}$ C on a hot day. Suddenly, the temperatures of both the large surfaces are reduced to 30 $^{\circ}$ C for all *t*>0 where *t* is the time in seconds. Develop an analytical expression for the transient temperature distribution *T*(*x*,*t*) in the porous slab. Assume the effective density of the slab is 850 kg/m³ and the effective specific heat is 1900 J/kg·K.

Equations

$$\frac{\partial}{\partial \mathbf{x}} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho \cdot c_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 x}{\partial x^2} + \lambda^2 x = 0 \quad ; \quad solution : \quad X(x) = A \quad cos(\lambda x) \\ + B \quad din(\lambda x) \\ \frac{\partial^2 x}{\partial x^2} - \lambda^2 \quad x = 0 \quad ; \quad solution : \quad X(x) = A \quad e^{-\lambda x} + B \quad e^{\lambda x} \\ \frac{\partial x^2}{\partial x^2} + c\lambda^2 \quad x = 0 \quad ; \quad solution : \quad X(t) = D \quad e^{-c\lambda^2 t} \\ \frac{\partial x}{\partial t} + c\lambda^2 \quad x = 0 \quad ; \quad solution : \quad X(t) = D \quad e^{-c\lambda^2 t} \\ \frac{\partial x}{\partial t} = \frac{1 - cos(2x)}{2}$$

Fall 2019 Qualifying Exam – Radiation Heat Transfer (Q2) – Closed Book

Real Body Radiation

Two parallel plates 0.5 m X 1 m are spaced 0.5 m apart. One plate is maintained at 1000 °C and the other at 500 °C. The plates have emissivities of 0.2 and 0.5 respectively and are located in a very large room whose walls are at 27 °C. The surfaces of the plates that are facing each other (surfaces 1 and 2) are exchanging heat between them and the room (surface 3). The backsides of the plates are thoroughly insulated. Find the net heat transfer rate to each plate and to the room.



Gas Mixtures

- (a) A mixture of 60% N_2 , 30% Ar and 10% O_2 on a mass basis is in a cylinder at 250 kPa, 310 K and volume 0.5 m³. Find the mole and the mass fractions and the mass of argon.
- (b) A slightly oxygenated air mixture is 69% N₂, 1% Ar and 30% O₂ on a mole basis. Assume a total pressure of 101 kPa and find the mass fraction of oxygen and its partial pressure.

$$mf_{i} = \frac{m_{i}}{m_{m}} \text{ and } y_{i} = \frac{N_{i}}{N_{m}} \qquad M_{m} = \frac{m_{m}}{N_{m}} = \sum_{i=1}^{k} y_{i}M_{i} \text{ and } R_{m} = \frac{R_{u}}{M_{n}}$$
$$m_{m} = \sum_{i=1}^{k} m_{i} \text{ and } N_{m} = \sum_{i=1}^{k} N_{i} \qquad mf_{i} = y_{i}\frac{M_{i}}{M_{m}} \text{ and } M_{m} = \frac{1}{\sum_{i=1}^{k}\frac{mf_{i}}{M_{i}}}$$

$$\frac{P_i}{P_m} = \frac{V_i}{V_m} = \frac{N_i}{N_m} = y_i$$

 $R_m = \Sigma c_i \cdot R_i$

Pv = RT

 $R_u = 8.31447 \text{ kJ/kmol} \cdot \text{K}$

Molar mass, gas constant, and critical-point properties

Substance	Formula	Molar mass, <i>M</i> kg/kmol	Gas constant, <i>R</i> kJ/kg · K*
Air	_	28.97	0.2870
Ammonia	NH ₂	17.03	0.4882
Argon	Ar	39.948	0.2081
Benzene	C ₆ H ₆	78.115	0.1064
Bromine	Br ₂	159.808	0.0520
<i>n</i> -Butane	$C_4 H_{10}$	58.124	0.1430
Carbon dioxide	CO ₂	44.01	0.1889
Carbon monoxide	CO	28.011	0.2968
Carbon tetrachloride	CCI4	153.82	0.05405
Chlorine	Cl _z	70.906	0.1173
Chloroform	CHCl ₃	119.38	0.06964
Dichlorodifluoromethane (R-12)	CCI ₂ F ₂	120.91	0.06876
Dichlorofluoromethane (R-21)	CHCl ₂ F	102.92	0.08078
Ethane	C ₂ H ₆	30.070	0.2765
Ethyl alcohol	C ₂ H ₅ OH	46.07	0.1805
Ethylene	C ₂ H ₄	28.054	0.2964
Helium	He	4.003	2.0769
<i>n</i> -Hexane	C ₆ H ₁₄	86.179	0.09647
Hydrogen (normal)	Hz	2.016	4.1240
Krypton	Kr	83.80	0.09921
Methane	CH ₄	16.043	0.5182
Methyl alcohol	CH3OH	32.042	0.2595
Methyl chloride	CH ₃ CI	50.488	0.1647
Neon	Ne	20.183	0.4119
Nitrogen	Nz	28.013	0.2968
Nitrous oxide	N _z O	44.013	0.1889
Oxygen	Oz	31.999	0.2598
Propane	C ₃ H ₈	44.097	0.1885
Propylene	C ₃ H ₆	42.081	0.1976
Sulfur dioxide	SOz	64.063	0.1298
Tetrafluoroethane (R-134a)	CF ₃ CH ₂ F	102.03	0.08149
Trichlorofluoromethane (R-11)	CCI ₃ F	137.37	0.06052
Water	H ₂ O	18.015	0.4615
Xenon	Xe	131.30	0.06332

Fall 2019 Qualifying Exam – Convection Heat Transfer (Q4) – Closed Book

Laminar Flow through a Pipe

"Slug" flow in a pipe may be described as that flow in which the velocity is constant across the entire flow area of the tube, i.e., $u = u_0$ (constant). Obtain an expression for the heat-transfer coefficient and the Nusselt number in this type of flow with a constant-heat-flux condition maintained at the wall.

Equations

$$\vec{\alpha} \cdot \frac{1}{8} \cdot \frac{\partial}{\partial x} \left(\vec{x} \cdot \frac{\partial T}{\partial x} \right) = \mathcal{U} \cdot \frac{\partial T}{\partial 2}$$

$$T_{m} : Bulk Mean Temperature$$

$$OF$$

$$Mixing Cup Temperature$$

$$T_{m} = \frac{\oint g \cdot c_{p} \cdot \mathcal{U} \cdot T \cdot dA}{\oint g \cdot c_{p} \cdot \mathcal{U} \cdot dA}$$

$$h = \frac{-k \cdot (\partial T/\partial r)}{F} = R}{T_{W} - T_{m}}$$

$$\overline{NU_{p}} = \frac{\overline{h} \cdot D}{k}$$