Qualifying Exam, Fall 2019
Fluid Mechanics

* This is a closed-book test (with cheat sheets included), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #_____________, #____________, and #_____________ to be graded.

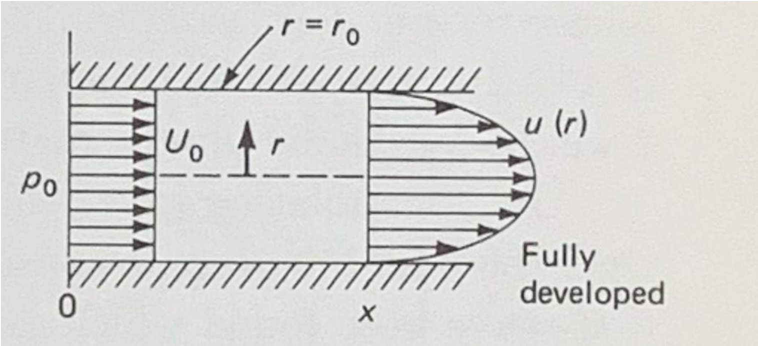
2. For laminar incompressible flow in the entrance to a pipe, as shown in the sketch below, the entrance flow is uniform, $u = U_0$, and the flow downstream at location x is parabolic in profile, $u(r) = C(r_0^2 - r^2)$, where r_0 is the radius of the pipe. Fluid density is ρ, pressure at section 0 and x are p_0 and p_x, respectively.

Find the expression of viscous drag force exerted by the pipe wall to the flow between section 0 and section x.

Hint: use control volume analysis.

Continuity equation: $\frac{\partial}{\partial t}\iiint_{CV} \rho dV + \oiint_{cs} \rho\vec{V}\cdot d\vec{A} = 0$

Momentum equation: $\frac{\partial}{\partial t}\iiint_{CV} \rho\vec{V} dV + \oiint_{cs} \vec{V}\rho\vec{V}\cdot d\vec{A} = \Sigma\vec{F}$



3. Consider the superposition of a sink with a counter-clockwise rotating point vortex (similar to flow towards bathtub sink).

a) Provide formula for stream function.

b) Provide formula for streamline (radius, r, as function of angular coordinate, φ). Have streamline originate from $r(\varphi=0)=1$.

c) What is ratio E/Γ for which radius is halved after each revolution, e.g. $r(2\pi)=0.5r(0)$?

d) Sketch streamline.

e) Provide formula for pressure coefficient, c_p, as function of radius, r. If you could not solve (c), use $E/\Gamma=(-\ln 0.5)/(2\pi)$. Use velocity at $r=1$ as reference velocity.

f) Plot pressure coefficient vs radius.

g) Develop formula for time that it takes for fluid particle to reach the origin (r=0).

Help:

Source

Potential: $\Phi = \frac{E}{2\pi}\ln\sqrt{x^2+y^2} = \frac{E}{2\pi}\ln r$

Stream function: $\Psi = \frac{E}{2\pi}\operatorname{atan}\frac{y}{x} = \frac{E}{2\pi}\varphi$

Counter-clockwise point vortex

Potential: $\Phi = \frac{\Gamma}{2\pi}\operatorname{atan}\frac{y}{x} = \frac{\Gamma}{2\pi}\varphi$

Stream function: $\Psi = -\frac{\Gamma}{2\pi}\ln\sqrt{x^2+y^2} = -\frac{\Gamma}{2\pi}\ln r$

Velocities:

$$v_r = \frac{\partial\Phi}{\partial r} = \frac{1}{r}\frac{\partial\Psi}{\partial\varphi}$$

$$v_\varphi = \frac{1}{r}\frac{\partial\Phi}{\partial\varphi} = -\frac{\partial\Psi}{\partial r}$$

4. Consider the following approximation for an incompressible boundary layer profile:

$$u = \begin{cases} u_e\left[1-\left(\frac{y}{\delta}-1\right)^2\right] & y \le \delta \\ u_e & else \end{cases}$$

a) What is the velocity gradient at the wall (*y*=0)?

b) Sketch profile. Include straight line for slope of boundary layer at the wall.

c) What is the skin-friction coefficient, $c_f(Re_\delta)$?

d) What is the displacement thickness?

e) What is the momentum thickness?

f) What is the shape factor? How does it compare to the shape factor for a Blasius boundary layer?

g) Assume that the boundary layer thickness varies according to $\delta = 4.93\sqrt{\frac{\nu x}{u_e}}$. Based on continuity equation, $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$, develop expression for wall-normal velocity (you should find $\frac{v}{u_e}=\frac{4.93}{6}\frac{1}{\sqrt{Re_x}}$).

h) Provide formula, *y(x)*, for streamline that passes through a downstream point, δ(*x*=*L*) (i.e. boundary layer edge at *x*=*L*). Compare with result from (d).

<u>Help:</u>

$Re_\delta = \frac{u_e\delta}{\nu}$ and $Re_x = \frac{u_e x}{\nu}$