Qualifying Exam, Fall 2019
Control

* This is a closed-book test (with cheat sheets included), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #____________, #___________, and #____________ to be graded.

Problem 1. A controlled process is modeled by the following state equations

$$\dot{x}_1 = 3x_1 + 7x_2, \quad \dot{x}_2 = 4x_1 + 5x_2 + u.$$

The control u is obtained from state feedback, such that

$$u = -k_1 x_1 - k_2 x_2$$

where k_1 and k_2 are real constants.

(a) Check controllability of the system (5 points)

(b) Determine the region in the k_1 versus k_2 parameter plane in which the closed-loop system is asymptotically stable. (5 points)

Problem 2.

(a) You are working for a company that tests 1-dimensional motion of a new car. Your task is measuring sensor data related to the motion of the car. However, because of the limited budget of your team, your boss asks you to select only one sensor between a position sensor and a velocity sensor. What is your choice? Why do you select that? Explain it using the observability concept with your computation. (6 points)

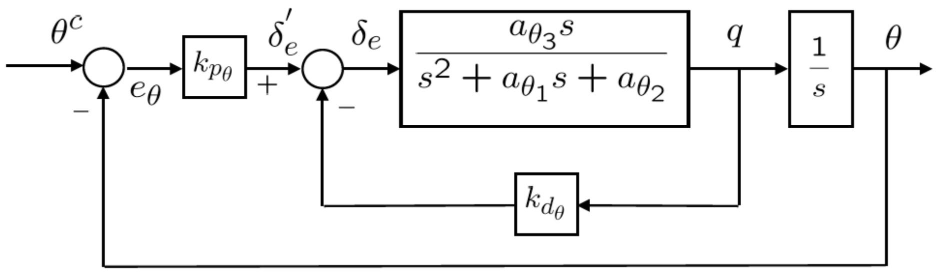
(b) Check the controllability and observability of the following system. (4points)

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x.$$

Problem 3. Consider the closed-loop PD control system for the pitch angle (θ) and pitch rate (q) of an airplane, as shown in the figure below.

(1) Find the transfer function from θ_c to θ. (6 pts)

(2) Given $k_{p_\theta} = 2, k_{d_\theta} = 0.6, a_{\theta_1} = 5, a_{\theta_2} = 1, and\ a_{\theta_3} = 7$, find the zeros and poles of the transfer function. (4 pts)



Problem 4. Consider a one degree-of-freedom robotic arm, whose dynamics is described by the simplified differential equation

$$\ddot{\theta} = 2u + \dot{\theta} - 5\theta,$$

where u is the input torque, and θ is the angle of the arm.

(1) Assuming that the measurement output is $y = \dot{\theta}$, find the state-space representation given the state definition $x = (\theta, \dot{\theta})$. (5 pts)

(2) Find the transfer function $\frac{\mathrm{Y(s)}}{U(s)}$ using the state-space representation. (5 pts)