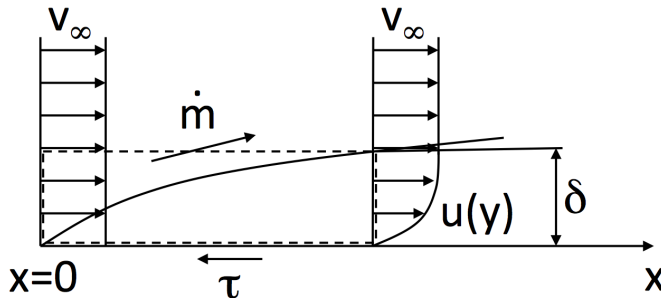


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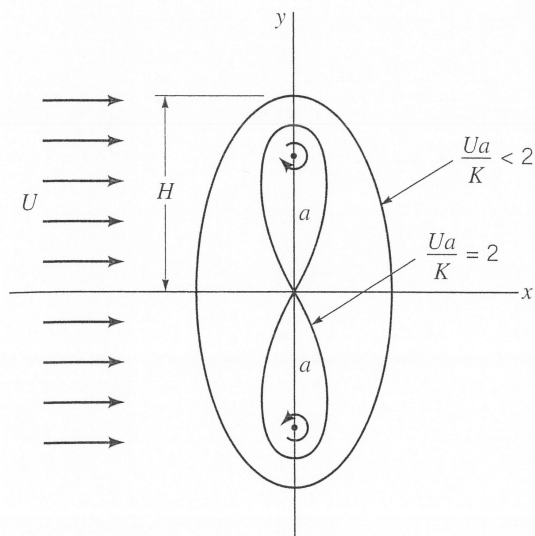
Fluid Mechanics Problems for Qualifying Exam (Fall 2014)

1. Consider a steady, incompressible boundary layer with thickness, $\delta(x)$, that develops on a flat plate with leading edge at $x = 0$. Based on a control volume analysis for the dashed box, answer the following:

- a) Provide an expression for the mass flux \dot{m} based on ρ , V_∞ , and δ .
- b) Provide an expression for the wall shear stress, τ_w , based on ρ , V_∞ , u and δ .
- c) Consider a linear velocity profile, $u(y)$. Provide an expression for τ_w as a function of ρ , V_∞ and $d\delta/dx$.
- d) The expression from (c) by definition equals $\tau_w = \mu\partial u/\partial y$ at $y = 0$. Provide an expression for $\delta(x)$ based on $Re_x = \rho V_\infty x/\mu$ and x .
- e) Provide the skin friction coefficient, $c_f = \frac{\tau_w}{0.5\rho V_\infty^2}$, as a function of Re_x .
- f) Based on the boundary layer shape, do you expect the skin friction coefficient for the Blasius boundary layer to be larger or smaller than your answer from (e)?

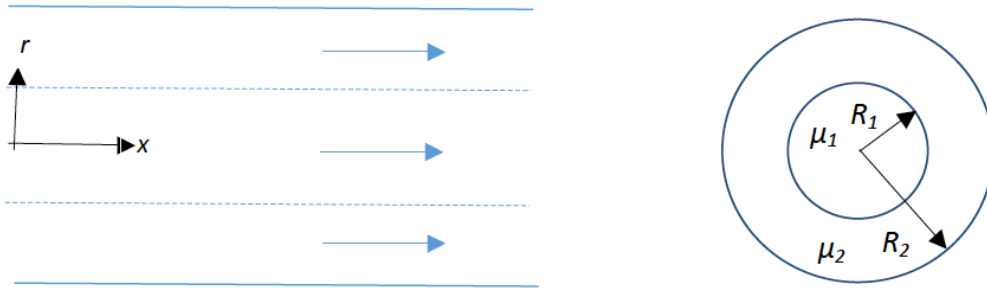


2. Consider a uniform flow with velocity U in the positive x direction combined with two free vortices of equal strength located along the y -axis. Let one vortex located at $y = a$ be a clockwise vortex ($\psi = K \ln r$) and the other at $y = -a$ be a counterclockwise vortex. It can be shown by plotting streamlines that for $Ua/K < 2$ the streamline $\psi = 0$ forms a closed contour, as shown in the figure below. Thus, this combination can be used to represent flow around a family of bodies called *Kelvin ovals*. For the case of $U = 10$ m/s, $a = 1$ m, if the body has half-height $H = 1.5$ m: a) calculate the value of K ; b) for the same Kelvin oval, calculate its maximum width along x .



3. Consider the steady, viscous, axisymmetric flow of two co-flowing fluids of viscosities μ_1 and μ_2 along the axis of a pipe of circular cross-section, as shown in the figure. The densities of both fluids are constant and equal and the fluids are driven by the same constant pressure gradient.

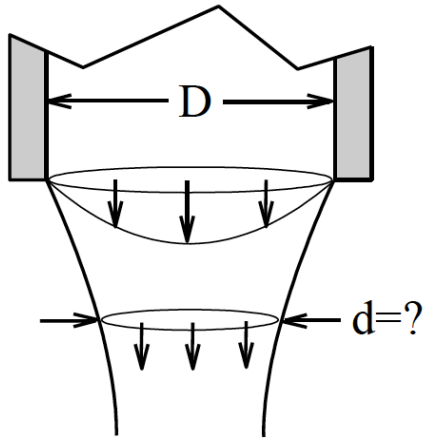
- (i) Making the parallel flow assumption, obtain the velocity distribution of this flow
- (ii) From your answer to (i), evaluate the velocity along the center line of the pipe



4. A viscous liquid steadily exits a circular vertical pipe (with inner diameter = D) with a parabolic velocity distribution (see figure). After the fluid exits the tube, viscous forces smooth the velocity profile to a uniform value. This happens in a very short distance from the exit, so that gravity force is negligible. Using control volume analysis (equations given below), find the diameter of the jet when the uniform flow is first established.

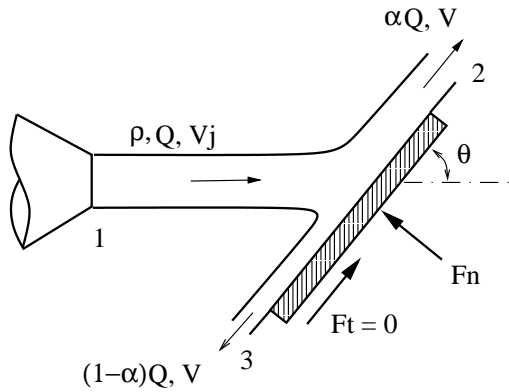
$$\frac{\partial}{\partial t} \int \int \int_{CV} \rho dV + \int \int_{CS} \rho(\mathbf{V} \cdot \mathbf{n}) dA = 0,$$

$$\frac{\partial}{\partial t} \int \int \int_{CV} \rho \mathbf{V} dV + \int \int_{CS} \rho \mathbf{V}(\mathbf{V} \cdot \mathbf{n}) dA = \Sigma(\mathbf{F}_{ext})_{C.V.},$$



Fluid Mechanics --- Fall 2013
(solve only 3 of 4 problems!!)

1. When an incompressible jet strikes an inclined fixed plate at speed V_j , as shown below, it breaks into two jets at 2 and 3 of equal velocity $V = V_j$ but unequal fluxes αQ at section 2 and $(1 - \alpha)Q$ at section 3, α being a fraction and Q being the total volume flowrate. The reason is that for frictionless flow the fluid can exert NO tangential force F_t on the plate. The condition $F_t = 0$ enables us to solve for α . Perform this analysis and find α as a function of the plate angle θ .



• $\alpha = \text{-----}$

Equation sheet for problem 2:

Continuity equation for coordinate (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho v_z) = 0$$

Momentum equation for coordinate (x, y, z):

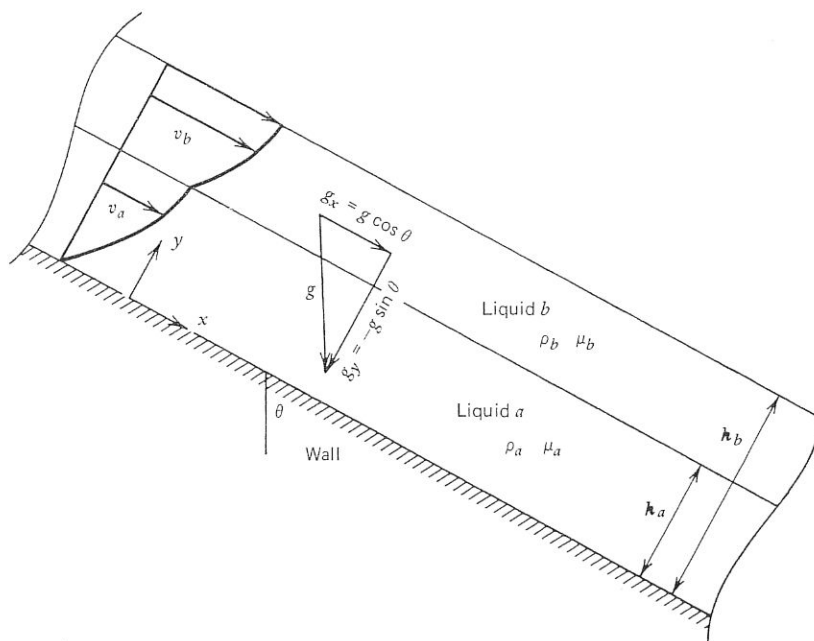
$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = \left(\frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial y} \tau_{yx} + \frac{\partial}{\partial z} \tau_{zx} \right) - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = \left(\frac{\partial}{\partial x} \tau_{xy} + \frac{\partial}{\partial y} \tau_{yy} + \frac{\partial}{\partial z} \tau_{zy} \right) - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = \left(\frac{\partial}{\partial x} \tau_{xz} + \frac{\partial}{\partial y} \tau_{yz} + \frac{\partial}{\partial z} \tau_{zz} \right) - \frac{\partial p}{\partial z} + \rho g_z$$

2. Consider a film consisting of two immiscible liquids, each of constant density and viscosity, flowing down an inclined plane, as shown in the figure. The flow is 2D, steady and parallel. Assume constant atmospheric pressure acts on the free surface of the top liquid.

- (a) Write down all the boundary conditions for this flow
- (b) Obtain expressions for the velocity fields v_a and v_b in the two liquids.



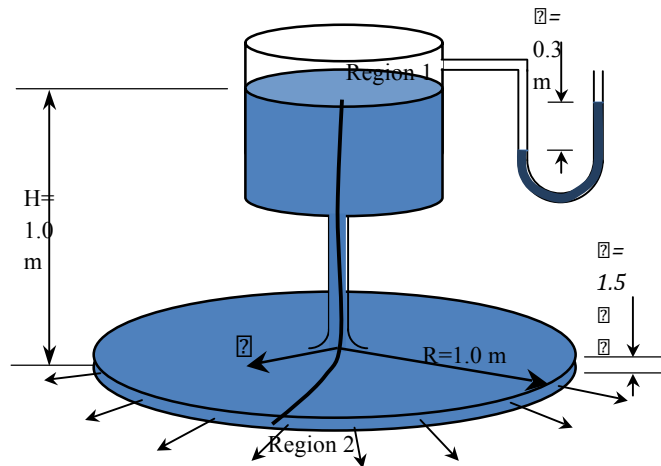
3. For the 2D, steady boundary layer developed when a fluid of density ρ and freestream speed U_∞ flows over a flat plate (with zero pressure gradient), show how to obtain the following equation

$$\tau_w = \rho \frac{d}{dx} \int_0^\delta u (U_\infty - u) dy,$$

where τ_w is the wall shear stress, u is the x -component of velocity and δ is the boundary layer thickness. Here, x denotes distance from the leading edge measured parallel to the plate and y is distance perpendicular to the plate.

4. Consider the tank and disk flow system shown in the figure below. Flow discharges to air through the thin gap between the two disks. Water in the tank is pressurized. Gauge fluid is also water. All dimensions are shown in the figure. Neglect water level change in the tank and the viscous effect.

- Find: (a) (3points) Air pressure in region 1
 (b) (3 points) Flow speed exiting the edge of the disks
 (c) (4 points) Pressure distribution inside the disks: $p(r)$



Solution:

- (a) $p_1 = \gamma_{\text{water}} h = 2.94 \text{ kPa (g)}$
- (b) Consider a streamline from water surface in the tank to the exit at the edge of the disk, as shown in the figure. Bernoulli's equation can be used:

$$p_1 + \frac{1}{2} \rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \gamma z_2$$
 Because the water level change in the tank can be neglected, thus $V_1 \cong 0 \rightarrow$

$$\frac{1}{2} \rho V_2^2 = (p_1 - p_2) + \gamma(z_1 - z_2) = \rho g(h + H)$$

$$\rightarrow V_2 = \sqrt{2g(h + H)} = 5.04 \text{ m/s}$$
- (c) Because the flow is incompressible, the flow rate across any cylindrical surface with radius r can be written as $Q = 2\pi r \delta V(r) = 2\pi R \delta V_2 \rightarrow V(r) = RrV_2$
 Along the streamline, the Bernoulli's equation is:

$$p_1 + \gamma z_1 = p(r) + \frac{1}{2} \rho V^2(r) + \gamma z_2$$

$$\rightarrow p(r) = p_1 + \gamma(z_1 - z_2) - \frac{1}{2} \rho V^2(r) = \rho g(h + H) - \frac{1}{2} \rho V_2^2 R^2 r^2 = \gamma(h + H)(1 - R^2 r^2) \text{ (gauge)}$$
 The pressure is negative, which means the pressure force intend to make the two disks closer to each other.

Fluid Mechanics Qualifying Exam

Study Material

The candidate is expected to have a thorough understanding of undergraduate engineering fluid mechanics topics. These topics are listed below for clarification. Not all instructors cover exactly the same material during a course, thus it is important for the candidate to closely examine the subject areas listed below. The textbooks listed below are a good source for the review and study of a majority of the listed topics. One final note, the example problems made available to the candidates are from past exams and do not cover all subject material. These problems are not to be used as the only source of study material. The topics listed below should be your guide for what you are responsible for knowing.

Suggested textbook:

Introduction to Fluid Mechanics, 4th Ed., Robert W. Fox and Alan T. McDonald, (John Wiley & Sons, pub.)

Fluid Mechanics, 3rd Ed., Frank M. White, (McGraw Hill, pub.)

Fluid Flow, 4th Ed., Rolf Sabersky, Allan Acosta, Edward Hauptmann, and E.M. Gates, (Prentice Hall, pub.)

Fundamentals of Fluid Mechanics, 4th Ed., Bruce R. Munson, Donald F. Young, and Theodore H. Okiishi, (John Wiley & Sons, pub.)

Topic areas:

1. Fluid properties
 - a. Viscosity
 - b. Compressibility
 - c. Surface tension
 - d. Ideal Gas Law
2. Fluid statics
 - a. Hydrostatic pressure
 - b. Forces and moments on solid surfaces
 - c. Manometers
3. Kinematics of fluid motion
 - a. Streamlines, pathlines, and streaklines
 - b. Local, convective and total derivative
 - c. Stream function and vorticity
 - d. Eulerian and Lagrangian descriptions
 - e. System and control volume
4. Bernoulli's Equation
 - a. For steady, inviscid and incompressible flows
 - b. Extension to other cases
5. Conservation laws in both differential and integral form
 - a. Continuity
 - b. Momentum (Navier-Stokes equations)
 - c. Energy
6. Simplified forms and their limitations
 - a. Euler's equation

- b. Laplace's equation
7. Similitude
 - a. Buckingham Pi Theorem
 - b. Dimensional analysis
 - c. Application to correction and modeling
 8. 2-D potential flow theory
 - a. Definition of potential flow
 - b. Linear superposition
 - c. Basic potential flow elements
 9. Fully developed pipe and duct flow
 - a. Laminar and turbulent flow solution methods
 - b. Moody diagram
 10. External flow
 - a. Boundary layer approximations, displacement and momentum thickness
 - b. Boundary layer equations, differential and integral
 - c. Flat plate solution
 - d. Lift and drag over bodies and use of lift and drag coefficients
 11. Basic 1-D compressible fluid flow
 - a. Speed of sound
 - b. Isentropic flow in duct of variable area
 - c. Normal shock waves
 - d. Use of tables to solve problems in above areas
 12. Non-dimensional numbers, their meaning and use
 - a. Reynolds number
 - b. Mach number
 - c. Euler number
 - d. Froude number
 - e. Prandtl number

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

1. The graph below depicts the drag coefficient for a sphere as a function of Reynolds number. A similar relationship is obtained for a very long cylinder in cross flow. Please discuss the various regions as to the flow phenomena taking place. In particular, explain the sudden decrease in drag observed around $Re = 300,000$

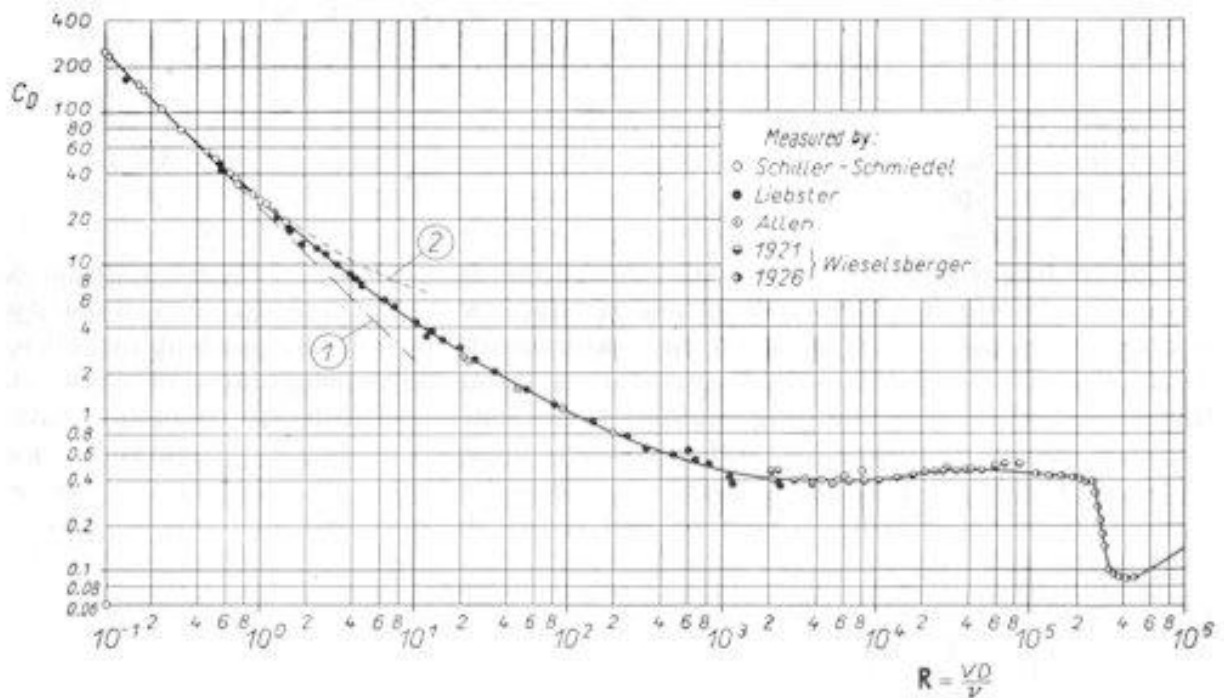


Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number
 Curve (1): Stokes' theory, eqn. (6.10); curve (2): Oseen's theory, eqn. (6.13)

Qualifying Exam: Fluid Mechanics**CLOSED BOOK**

2. A flow satisfies the following two conditions: $\vec{\nabla} \cdot \vec{V} = 0$ and $\vec{\nabla} \times \vec{V} = 0$

a. What does each of these equations represent?

b. Given a 2-D flow described by the potential function

$$\phi(r, \theta) = \frac{\kappa}{2\pi} \frac{\cos(\theta)}{r}$$

Determine if the above conditions are satisfied. You may use the following equations:

$$x = r \sin(\theta) \quad y = r \cos(\theta)$$

$$\vec{\nabla} = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \quad \text{which is the "del" operator}$$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

Qualifying Exam: Fluid Mechanics**CLOSED BOOK**

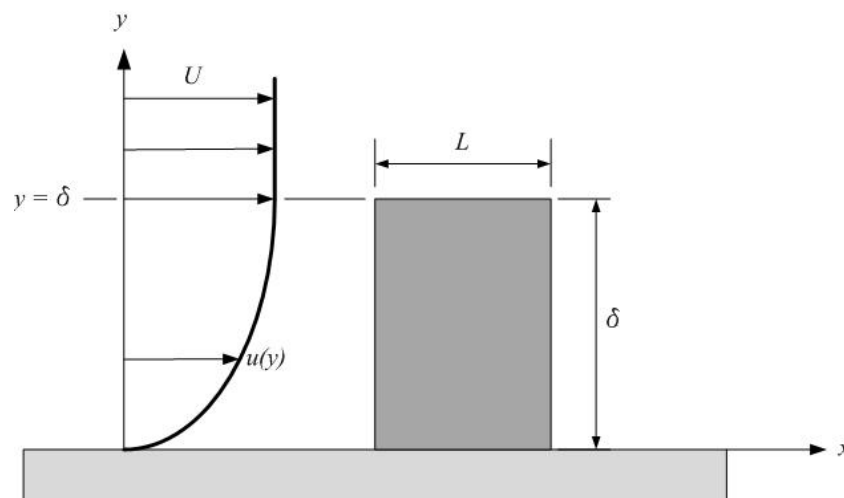
3. A flat plate of length L and height δ is placed at a wall and is parallel to an approaching wall boundary layer, as shown in the figure below. Assume that there is no flow in the y direction and that in any plane $y = \text{constant}$, the boundary layer that develops over the plate is the Blasius solution for a flat plate. If the approaching wall boundary layer has a velocity profile approximated by:

$$u(y) = U \left[\sin\left(\frac{\pi y}{2\delta}\right) \right]^{\frac{2}{3}}$$

Find an expression for the drag force on the plate. Recall the transformation of variables in the Blasius problem:

$$\eta = \left(\frac{U_e}{2\nu x} \right)^{\frac{1}{2}} z \quad \text{and} \quad u = U_e f'(\eta)$$

Where U_e is the velocity at the edge of the boundary layer and z is the coordinate normal to the plate. Further, $f''(0) = 0.4696$.



Qualifying Exam: Fluid Mechanics

CLOSED BOOK

4. The momentum and energy equations, in tensor notation, for the Raleigh-Benard problem are as follows:

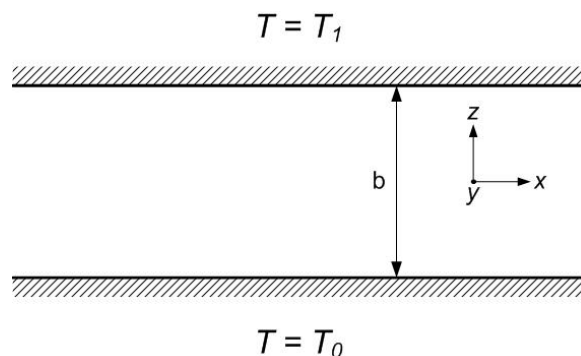
$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} - g[1 - \beta(T - T_0)]\delta_{i3} + \nu \nabla^2 u_i,$$

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \alpha \nabla^2 T,$$

With $i, j = 1, 2, 3$ corresponding to the $x, y,$ and z directions, respectively, and δ_{i3} is the Kronecker delta. The variables are the velocity components, u_i , the pressure, p , the temperature, T . The different parameters in the equations are: α , the coefficient of thermal diffusivity, β , the coefficient of thermal expansion, ν , the coefficient of kinematic viscosity, and ρ_0 , a reference density. These equations apply to the fluid trapped between two parallel rigid walls maintained at fixed temperatures, T_0 (lower wall) and T_1 (upper wall, with $T_0 > T_1$, see figure below). Assume that the fluid extends to infinity in the x and y directions. These equations are of course coupled with the continuity equations for incompressible flows.

- Assuming that the base state is one in which the fluid is at rest and the flow steady everywhere, find the temperature and pressure distributions, $\bar{T}(z)$ and $\bar{p}(z)$, respectively, in the base state (since wall pressures are not specified, you can leave any constants of integration in the pressure distribution as they are.)
- Write down the linearized disturbance equations.

(Note: You are not required to know anything about the Raleigh-Benard problem to be able to answer these questions.)



Qualifying Exam: Fluid Mechanics**CLOSED BOOK**

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

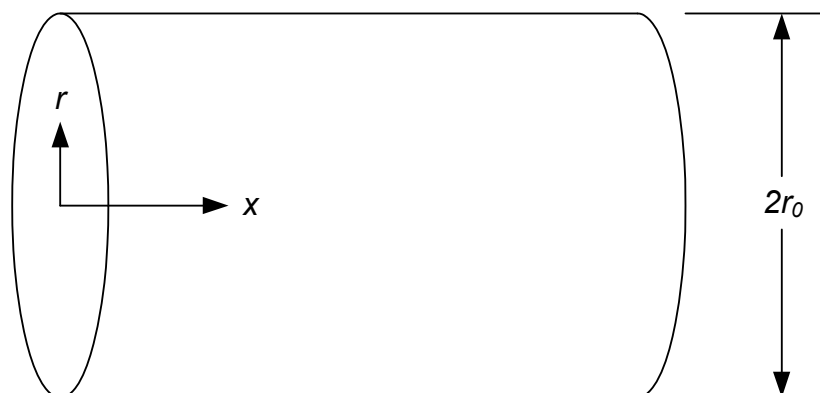
1. Discuss the requirements for accurate fluid mechanical testing of models, such as models of aircraft and cars. What are the requirements? What are the practical limitations? Use dimensionless parameters to help explain. Be sure to identify what these dimensionless parameters represent.
2. The continuity and momentum equations for 2-D flow for a cylindrical coordinate system are:

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial(rv)}{\partial r} = 0$$

$$u \frac{\partial u}{\partial x} + \frac{v}{r} \frac{\partial(ur)}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right]$$

$$u \frac{\partial v}{\partial x} + \frac{v}{r} \frac{\partial(vr)}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[\frac{\partial^2 v}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) \right]$$

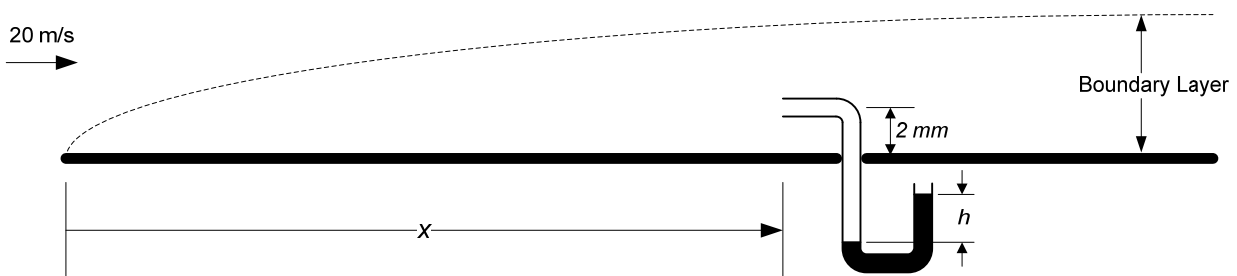
where u and v are velocity components in x and r directions, respectively. Simplify the above equations to obtain the momentum equation for hydrodynamically fully developed flow in a circular tube. Use the resulting equation and appropriate boundary condition to obtain velocity distribution for hydrodynamically fully developed flow in a circular tube. Find mean velocity u_m and express the velocity distribution in form $u(r)/u_m = f(r)$.



Qualifying Exam: Fluid Mechanics

CLOSED BOOK

3. Air at standard conditions flows past a smooth flat plate at 20 m/s , as shown below. A pitot stagnation tube with its center placed 2 mm above the plate develops a water manometer head of $h = 21 \text{ mm}$.
- Estimate the flow speed parallel to the plate at the location of the tube.
 - Assuming a laminar flat plate boundary layer, estimate the x position of the tube.



Note: $\rho_{air} = 1.16 \frac{\text{kg}}{\text{m}^3}$
 $\nu_{air} = 15.9 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$
 $\rho_{water} = 1000 \frac{\text{kg}}{\text{m}^3}$
 $\nu_{water} = 0.855 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

4. You are to use an integral control volume analysis to determine the laminar boundary layer thickness δ as a function of x . Consider the flow over a smooth flat plate of a Newtonian fluid, with no pressure gradient in the flow direction. As the solution is approximate, the choice of boundary layer velocity profile is somewhat open. The main physics, however, can be captured with even a crude choice such as

$$u(x, y) = u_\infty \frac{y}{\delta(x)} \quad 0 \leq y \leq \delta, \quad u = u_\infty \text{ for } y > \delta$$

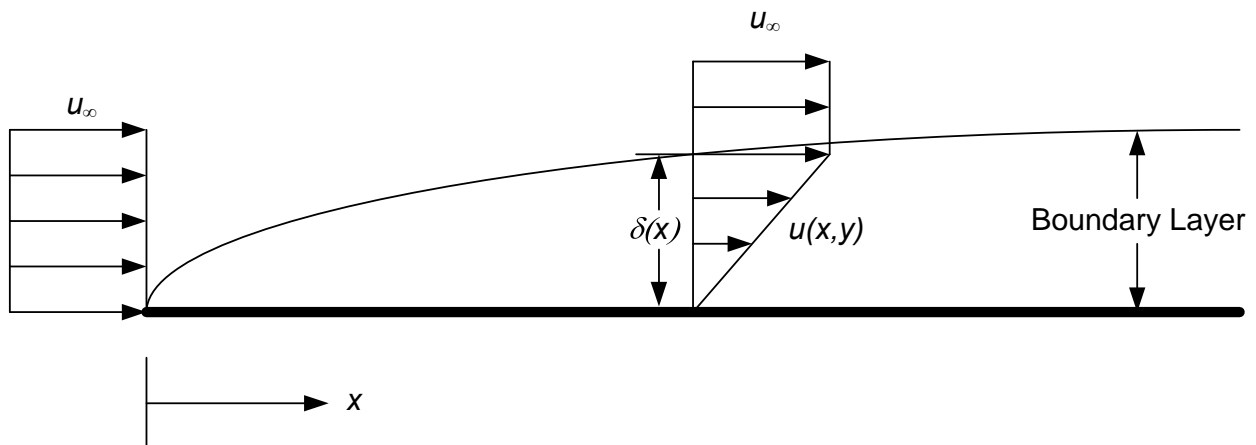
Use this simple profile. The properties viscosity, μ , and density, ρ , are to be taken as constant.

The steady-state control volume equations for x and y momentum can be written as:

$$F_x = F_{Sx} + F_{Bx} = \int_{CS} u \rho \bar{V} \cdot d\bar{A}$$

$$F_y = F_{Sy} + F_{By} = \int_{CS} v \rho \bar{V} \cdot d\bar{A}$$

Here S and B designate surface and body forces. CS is the control surface, while u and v are the x and y components of the velocity vector, \bar{V} .



The fluid may be assumed incompressible: $\int_{CS} \bar{V} \cdot d\bar{A}$

Name _____

Spring 2008

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

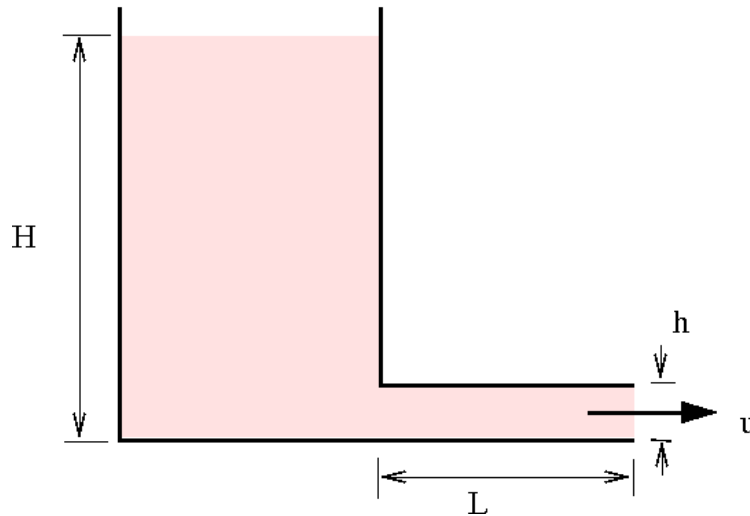
I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

1. Consider a Bingham plastic of density ρ draining from a reservoir through a two-dimensional channel (see figure below).



A Bingham plastic is a non-Newtonian fluid with the stress-strain relation

$$\tau_{ij} = \tau_0 + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad \text{for} \quad |\tau| \geq \tau_0.$$

For $|\tau| < \tau_0$, the fluid behaves like a rigid body ($\mu \rightarrow \infty$). Assume that μ and ρ are constant, that the reservoir is large, and that τ_0 is small enough that the fluid does flow.

- Use a global force balance to find the shear stress on the channel walls (assume pressure at the exit is equal to the ambient pressure).
 - Find the strain-rate at the channel walls.
 - Use the strain-rate (in addition to “no slip”) as a wall boundary condition to find the velocity profile in the channel. Be sure to specify the point away from the wall where the fluid begins to behave as a rigid body.
2. Draw one or more free body diagrams showing all the aero and hydrodynamic forces that control the movement of a sail boat when the sail boat is sailing 90 degrees to the direction of the wind. Discuss how these forces affect the motion of the sail boat.

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

3. Calculate the total kinetic energy of an Oseen vortex

$$v_{\theta} = \frac{\Gamma}{2\pi r} \left[1 - \exp\left(-\frac{r^2}{4\nu t}\right) \right]$$

and a Taylor vortex

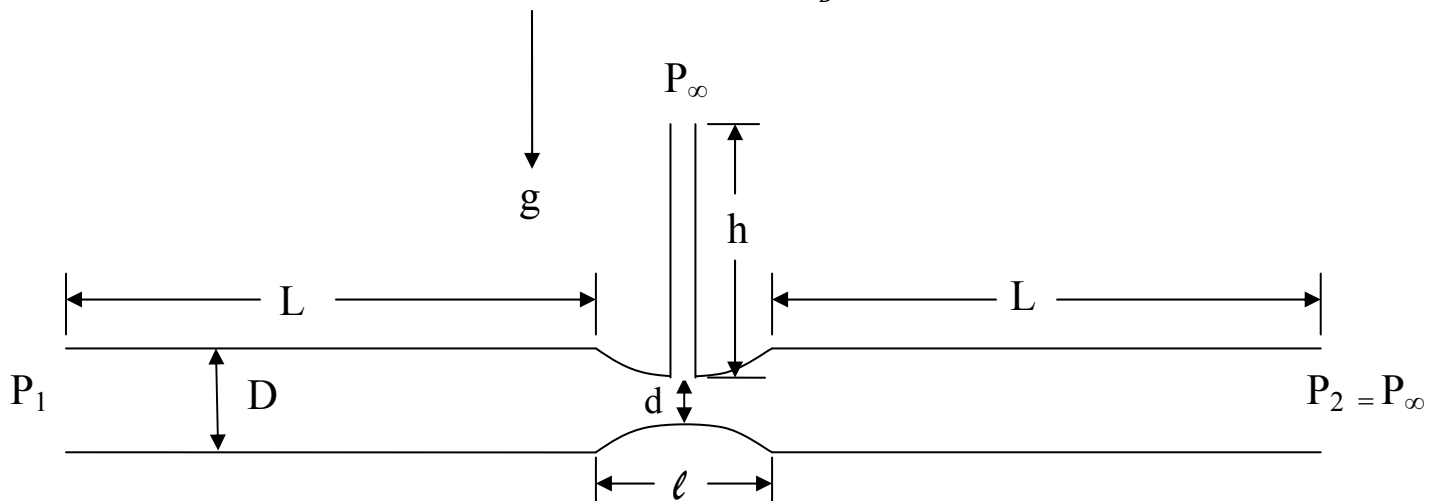
$$v_{\theta} = \frac{H}{8\pi} \frac{r}{\nu^2} \exp\left(-\frac{r^2}{4\nu t}\right).$$

Is the Taylor vortex a solution of the incompressible Navier-Stokes equations? Explain.

4. Consider the pipe setup in the figure below. The flow at 1 is fully developed and exits to atmosphere at 2. A venturi is placed halfway between 1 and 2. There is a small tube that rises vertically from the throat of the venturi. You are to determine the minimum height, h , of this tube such that no water exits out the top of it. You may use the following approximations: $L \gg \ell$, and the venturi may be treated as frictionless. The pipe wall itself is smooth. Use the following values and the additional sheets provided.

$$D = 4 \text{ cm}, d = 2 \text{ cm}, L = 50 \text{ m}, \rho = 1000 \text{ kg/m}^3, P_1 = 120 \text{ kPa}, P_2 = P_{\infty} = 100 \text{ kPa}$$

$$g = 9.81 \text{ m/s}^2, \mu = 0.001 \text{ N-s/m}^2. \text{ You may use } f = \frac{0.3164}{Re_D^{0.25}} \text{ for } Re_D < 10^5.$$



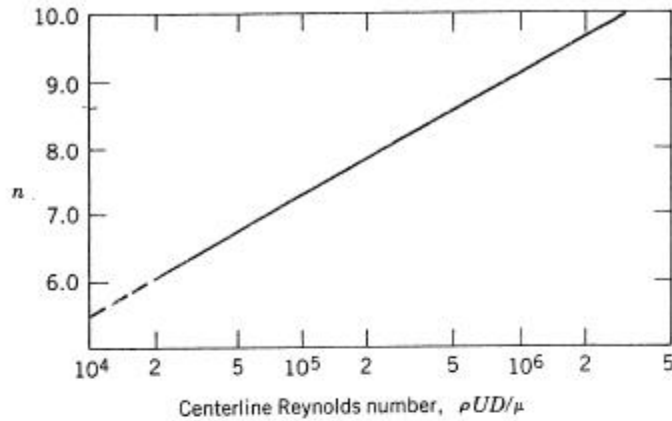


Fig. 8.11 Exponent for power-law profile. (Adapted from [5].)

The power-law profile is not applicable close to the wall ($y/R < 0.04$); the profile gives infinite velocity gradient at the wall. Although the profile fits the data close to the centerline, it fails to give zero slope at the centerline. The variation of exponent n in the power-law profile with Reynolds number (based on pipe diameter, D , and centerline velocity, U) is shown in Fig. 8.11.

Since the average velocity is $\bar{V} = Q/A$, and

$$Q = \int_A \vec{V} \cdot d\vec{A}$$

the ratio of the average velocity to the centerline velocity may be calculated for the power-law profiles of Eq. 8.22. The result is

$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} \quad (8.23)$$

From Eq. 8.23, we see that as n increases (due to increasing Reynolds number) the ratio of the average velocity to the centerline velocity increases; with increasing Reynolds number the velocity profile becomes more blunt or “fuller” (for $n = 6$, $\bar{V}/U = 0.79$; for $n = 10$, $\bar{V}/U = 0.87$). As a representative value, 7 often is used for the exponent; this gives rise to the term “a one-seventh power profile” for fully developed turbulent flow.

Velocity profiles for $n = 6$ and $n = 10$ are shown in Fig. 8.12. The parabolic profile for fully developed laminar flow has been included for comparison. It is clear that the turbulent profile has a much steeper slope near the wall.

8-6 ENERGY CONSIDERATIONS IN PIPE FLOW

Thus far in our discussion of viscous flow, we have derived all results by applying the momentum equation for a control volume. We have, of course, also used the control volume formulation of conservation of mass. Nothing has been said about conservation of energy—the first law of thermodynamics. Additional insight into the nature of the pressure losses in internal viscous flows can be obtained from the energy equation. Consider, for example, steady flow through the piping system, including

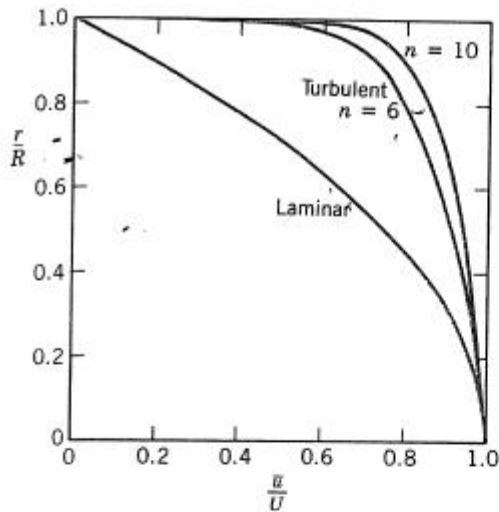


Fig. 8.12 Velocity profiles for fully developed pipe flow.

a reducing elbow, shown in Fig. 8.13. The control volume boundaries are shown as dashed lines. They are normal to the flow at sections ① and ② and coincide with the inside pipe wall elsewhere.

Basic equation:

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{\text{CV}} e \rho \, dV + \int_{\text{CS}} (e + pv) \rho \vec{V} \cdot d\vec{A} \quad (4.57)$$

$$e = u + \frac{V^2}{2} + gz$$

- Assumptions:
- (1) $\dot{W}_s = 0, \dot{W}_{\text{other}} = 0$
 - (2) $\dot{W}_{\text{shear}} = 0$ (although shear stresses are present at the walls of the elbow, the velocities are zero at the walls)
 - (3) Steady flow
 - (4) Incompressible flow
 - (5) Internal energy and pressure uniform across sections ① and ②

Under these assumptions the energy equation reduces to

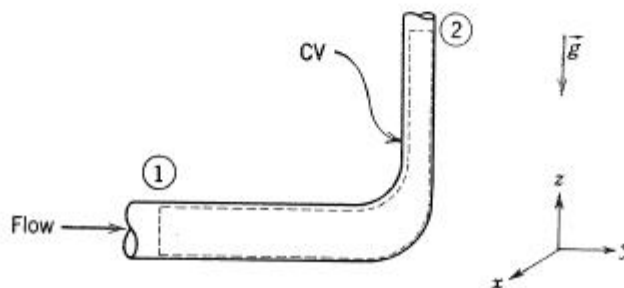


Fig. 8.13 Control volume and coordinates for energy analysis of flow through a 90° reducing elbow.

$$\begin{aligned} \dot{Q} = \dot{m}(u_2 - u_1) + \dot{m} \left(\frac{p_2}{\rho} - \frac{p_1}{\rho} \right) + \dot{m} g(z_2 - z_1) \\ + \int_{A_2} \frac{V_2^2}{2} \rho V_2 dA_2 - \int_{A_1} \frac{V_1^2}{2} \rho V_1 dA_1 \end{aligned} \quad (8.24)$$

Note that we have not assumed the velocity to be uniform at sections ① and ②, since we know for viscous flows the velocity at a section cannot be uniform. However, it is convenient to introduce the average velocity into Eq. 8.24 so that we can eliminate the integrals. To do this, we define a kinetic energy coefficient.

8-6.1 Kinetic Energy Coefficient

The *kinetic energy coefficient*, α , is defined such that

$$\int_A \frac{V^2}{2} \rho V dA = \alpha \int_A \frac{\bar{V}^2}{2} \rho V dA = \alpha \dot{m} \frac{\bar{V}^2}{2} \quad (8.25a)$$

or

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m} \bar{V}^2} \quad (8.25b)$$

For laminar flow in a pipe (velocity profile given by Eq. 8.12), $\alpha = 2.0$.

In turbulent pipe flow, the velocity profile is quite flat, as shown in Fig. 8.12. We can use Eq. 8.25b together with Eqs. 8.22 and 8.23 to determine α . Substituting the power-law velocity profile of Eq. 8.22 into Eq. 8.25b, we obtain

$$\alpha = \left[\frac{U}{\bar{V}} \right]^3 \frac{2n^2}{(3+n)(3+2n)} \quad (8.26)$$

The value of \bar{V}/U is determined from Eq. 8.23. For $n = 6$, $\alpha = 1.08$; for $n = 10$, $\alpha = 1.03$. Since the exponent, n , in the power-law profile is a function of Reynolds number, α also varies with Reynolds number. Because α is reasonably close to one for large Reynolds number, unity often is assumed for pipe flow calculations. However, for developing flows at moderate Reynolds numbers the change of kinetic energy may be significant.

8-6.2 Head Loss

Using the definition of α , the energy equation (Eq. 8.24) can be written

$$\dot{Q} = \dot{m}(u_2 - u_1) + \dot{m} \left(\frac{p_2}{\rho} - \frac{p_1}{\rho} \right) + \dot{m} g(z_2 - z_1) + \dot{m} \left(\frac{\alpha_2 \bar{V}_2^2}{2} - \frac{\alpha_1 \bar{V}_1^2}{2} \right)$$

Dividing by the mass rate of flow gives

$$\frac{\delta Q}{dm} = u_2 - u_1 + \frac{p_2}{\rho} - \frac{p_1}{\rho} + g z_2 - g z_1 + \frac{\alpha_2 \bar{V}_2^2}{2} - \frac{\alpha_1 \bar{V}_1^2}{2}$$

Rearranging this equation, we write

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1 \right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2 \right) = (u_2 - u_1) - \frac{\delta Q}{dm} \quad (8.27)$$

In Eq. 8.27, the term

$$\left(\frac{p}{\rho} + \alpha \frac{\bar{V}^2}{2} + gz \right)$$

represents the mechanical energy per unit mass at a cross section. The term $u_2 - u_1 - \delta Q/dm$ is equal to the difference in mechanical energy per unit mass between sections ① and ②. It represents the (irreversible) conversion of mechanical energy at section ① to unwanted thermal energy ($u_2 - u_1$) and loss of energy via heat transfer ($-\delta Q/dm$). We identify this group of terms as the total head loss, h_{l_T} . Then

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1 \right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2 \right) = h_{l_T} \quad (8.28)$$

Head loss has dimensions of energy per unit mass [FL/M]; this is equivalent to dimensions of [L^2/t^2].

If the flow were assumed frictionless, the velocity at a section would be uniform ($\alpha_1 = \alpha_2 = 1$) and Bernoulli's equation would predict zero head loss.

In incompressible frictionless flow, a change in internal energy can occur only through heat transfer; there is no conversion of mechanical energy ($p/\rho + V^2/2 + gz$) to internal energy. For viscous flow in a pipe, one effect of friction may be to increase the internal energy of the flow, as shown by Eq. 8.27.

You might wonder why the energy loss, h_{l_T} , is called a "head" loss. As the empirical science of hydraulics developed during the nineteenth century, it was common practice to express the energy balance in terms of energy per unit *weight* of flowing liquid (e.g., water) rather than energy per unit *mass*, as in Eq. 8.28. To obtain dimensions of energy per unit weight, we divide each term in Eq. 8.28 by the acceleration of gravity, g . Then the net dimensions of h_{l_T} are $[(L^2/t^2)(t^2/L)] = [L]$, or feet of flowing liquid. Since the term head loss is in common use, we shall also use it here. Remember that its physical interpretation is a loss in mechanical energy per unit mass of flowing fluid.

Equation 8.28 can be used to calculate the pressure difference between any two points in a piping system, provided the head loss, h_{l_T} , can be determined. We shall consider calculation of h_{l_T} in the next section.

8-7 CALCULATION OF HEAD LOSS

Total head loss, h_{l_T} , is regarded as the sum of major losses, h_l , due to frictional effects in fully developed flow in constant-area tubes, and minor losses, h_{l_m} , due to entrances, fittings, area changes, and so on. Consequently, we consider the major and minor losses separately.

8-7.1 Major Losses: Friction Factor

The energy balance, expressed by Eq. 8.28, can be used to evaluate the major head loss. For fully developed flow through a constant-area pipe, $h_{l_m} = 0$, and

$\alpha_1(\bar{V}_1^2/2) = \alpha_2(\bar{V}_2^2/2)$; Eq. 8.28 reduces to

$$\frac{p_1 - p_2}{\rho} = g(z_2 - z_1) + h_l \quad (8.29)$$

If the pipe is horizontal, then $z_2 = z_1$ and

$$\frac{p_1 - p_2}{\rho} = \frac{\Delta p}{\rho} = h_l \quad (8.30)$$

Thus the major head loss can be expressed as the pressure loss for fully developed flow through a horizontal pipe of constant area.

Since head loss represents the energy converted by frictional effects from mechanical to thermal energy, head loss for fully developed flow in a constant-area duct depends only on the details of the flow through the duct. Head loss is independent of pipe orientation.

a. Laminar Flow

In laminar flow, the pressure drop may be computed analytically for fully developed flow in a horizontal pipe. Thus, from Eq. 8.13c,

$$\Delta p = \frac{128\mu L Q}{\pi D^4} = \frac{128\mu L \bar{V}(\pi D^2/4)}{\pi D^4} = 32 \frac{L}{D} \frac{\mu \bar{V}}{D}$$

Substituting in Eq. 8.30 gives

$$h_l = 32 \frac{L}{D} \frac{\mu \bar{V}}{\rho D} = \frac{L}{D} \frac{\bar{V}^2}{2} \left(64 \frac{\mu}{\rho \bar{V} D} \right) = \left(\frac{64}{Re} \right) \frac{L}{D} \frac{\bar{V}^2}{2} \quad (8.31)$$

(We shall see the reason for writing h_l in this form shortly.)

b. Turbulent flow

In turbulent flow we cannot evaluate the pressure drop analytically; we must resort to experimental results and use dimensional analysis to correlate the experimental data. In fully developed turbulent flow, the pressure drop, Δp , due to friction in a horizontal constant-area pipe is known to depend on pipe diameter, D , pipe length, L , pipe roughness, e , average flow velocity, \bar{V} , fluid density, ρ , and fluid viscosity, μ . In functional form

$$\Delta p = \Delta p(D, L, e, \bar{V}, \rho, \mu)$$

We applied dimensional analysis to this problem in Example Problem 7.2. The results were a correlation of the form

$$\frac{\Delta p}{\rho \bar{V}^2} = f \left(\frac{\mu}{\rho \bar{V} D}, \frac{L}{D}, \frac{e}{D} \right)$$

We recognize that $\mu/\rho \bar{V} D = 1/Re$, so we could just as well write

$$\frac{\Delta p}{\rho \bar{V}^2} = \phi \left(Re, \frac{L}{D}, \frac{e}{D} \right)$$

Substituting from Eq. 8.30, we see that

$$\frac{h_l}{\bar{V}^2} = \phi \left(Re, \frac{L}{D}, \frac{e}{D} \right)$$

Although dimensional analysis predicts the functional relationship, we must obtain actual values experimentally.

Experiments show that the nondimensional head loss is directly proportional to L/D . Hence we can write

$$\frac{h_l}{\bar{v}^2} = \frac{L}{D} \phi_1 \left(Re, \frac{e}{D} \right)$$

Since the function, ϕ_1 , is still undetermined, it is permissible to introduce a constant into the left side of the above equation. The number $\frac{1}{2}$ is introduced into the denominator such that the head loss is nondimensionalized on the kinetic energy per unit mass of flow. Then

$$\frac{h_l}{\frac{1}{2} \bar{v}^2} = \frac{L}{D} \phi_2 \left(Re, \frac{e}{D} \right)$$

The unknown function, $\phi_2(Re, e/D)$, is defined as the *friction factor*, f ,

$$f = \phi_2 \left(Re, \frac{e}{D} \right)$$

and

$$h_l = f \frac{L}{D} \frac{\bar{v}^2}{2} \tag{8.32}$$

The friction factor³ is determined experimentally. The results, published by L. F. Moody [6], are shown in Fig. 8.14.

To determine head loss for fully developed flow with known conditions, the Reynolds number is evaluated first. Relative roughness, e/D , is obtained from Fig. 8.15. Then the friction factor, f , is read from the appropriate curve in Fig. 8.14, at the known values of Re and e/D . Finally, head loss is found using Eq. 8.32.

Several features of Fig. 8.14 require some discussion. The friction factor for laminar flow may be obtained by comparing Eqs. 8.31 and 8.32

$$h_l = \left(\frac{64}{Re} \right) \frac{L}{D} \frac{\bar{v}^2}{2} = f \frac{L}{D} \frac{\bar{v}^2}{2}$$

Consequently, for laminar flow

$$f_{\text{laminar}} = \frac{64}{Re} \tag{8.33}$$

Thus, in laminar flow, the friction factor is a function of Reynolds number only; it is independent of roughness. Although we took no notice of roughness in deriving Eq. 8.31, experimental results verify that the friction factor is a function only of Reynolds number in laminar flow.

The Reynolds number in a pipe may be changed most easily by varying the average flow velocity. If the flow in a pipe is originally laminar, increasing the velocity until the critical Reynolds number is reached causes transition to occur; the laminar flow

³ The friction factor defined by Eq. 8.32 is the *Darcy friction factor*. The *Fanning friction factor*, less frequently used, is defined in Problem 8.74.

Qualifying Exam: Fluid Mechanics

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This portion of the qualifying exam is **closed** book. You may have a calculator.

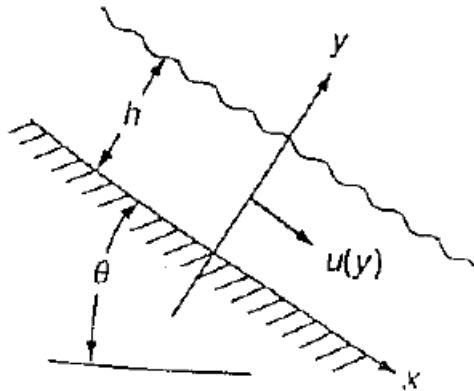
Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

1. Consider the incompressible flow of a fluid of viscosity μ down an inclined plane, as shown in the figure below. Assume that the flow is steady, one-dimensional (i.e. the only non-zero component of velocity is along the x-axis) and the atmosphere exerts constant pressure and negligible shear on the free surface. Derive an expression for $u(y)$. (Note: the figure is a cartoon, ignore the 'waves' you see on the surface).

FIGURE 1

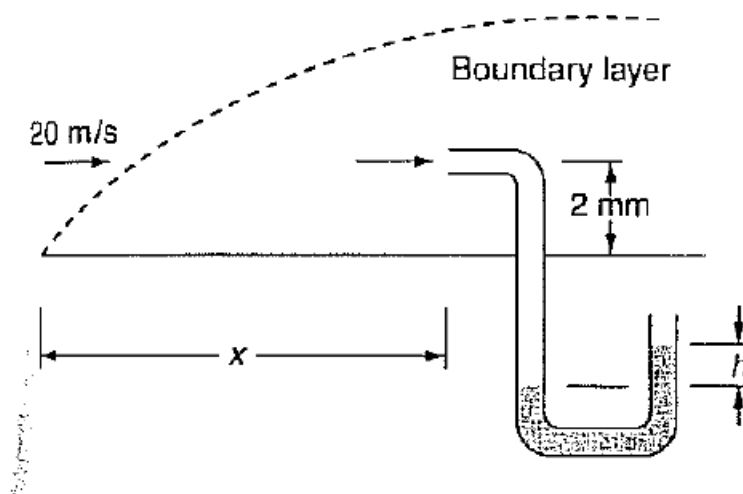


Qualifying Exam: Fluid Mechanics

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2. Air at standard conditions flows past a smooth flat plate, as in the Figure below. A pitot stagnation tube, placed 2 mm from the wall, develops a water manometer head $h = 21$ mm.
- Estimate the flow speed parallel to the plate at the location of the tube.
 - Assuming a laminar flat plate boundary layer, estimate the position x of the tube.

FIGURE 2



1. Navi

$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i, \quad i = 1, 2, 3$$

2. Continuity equation for Incompressible Flows

$$\frac{\partial u_i}{\partial x_i} = 0$$

3. Bernoulli's Equation

$$p + \frac{1}{2} \rho V^2 + \rho g z = \text{constant along a streamline}$$

4. Constitutive relation for Newtonian fluids

$$\tau_{ij} = -p \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

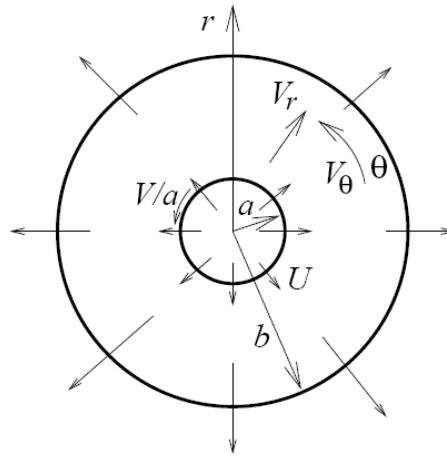
Qualifying Exam: Fluid Mechanics

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3.

Consider steady incompressible flow between two concentric cylinders shown below, with radii a and b , respectively. Liquid of constant kinematic viscosity ν fills the gap between the cylinders. The inner cylinder (radius a) rotates at a constant angular rate (V/a) and the outer cylinder (radius b) is fixed in space.

Liquid seeps through the inner cylinder radially at a constant velocity U ; fluid also seeps radially through the outer cylinder. ASSUME that flow parameters only depend on the radial distance r measured from the center of cylinder a .



(a) Show that the radial velocity component in the entire gap region is given by

$$v_r = \frac{aU}{r}$$

(b) Write down the governing equation for v_θ , the tangential velocity component in the gap, and derive the solution for $v_\theta(r)$ with the given boundary conditions.

(hint: 2D steady incompressible N-S equations in cylindrical coordinate

$$\frac{1}{r} \frac{\partial}{\partial r}(rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0$$

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} = \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r}(rv_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} \right] - \frac{\partial p}{\partial r}$$

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} = \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r}(rv_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right] - \frac{1}{r} \frac{\partial p}{\partial \theta}$$

)

Name _____

Spring 2007

Qualifying Exam: Fluid Mechanics

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4. Please discuss the various contributions to fluid dynamical drag, paying particular attention to the mechanisms and their relative contribution to total drag for the following situations.
 - a. Fully immersed object with Reynolds number less than one.
 - b. Fully immersed object with Reynolds number much greater than one.
 - c. Object moving at fluid interface such as a ship on the ocean.

Name _____

Fall 2006

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #___, #___, and #___ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

3. Consider a beaker of water in which are contained a few small bits of tea leaves that have absorbed water and sunk to the bottom. The fact that they are tea leaves is not important, rather what is important is that there are some small bits of matter that are somewhat denser than water.

Now a spoon or swizzle stick is use to vigorously stir the water in a circular fashion, causing the water to rotate more or less about the vertical axis of the beaker centerline. At first the tea leaves are dispersed, but are then observed to sink back to the bottom and migrate toward the center of the beaker bottom where they remain.

Please explain this behavior form a fluid mechanical standpoint.

Qualifying Exam: Fluid Mechanics

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4. You are to use an integral control volume analysis to determine the laminar boundary layer thickness δ as a function of x . Consider the flow over a smooth flat plate of a Newtonian fluid, with no pressure gradient in the flow direction. As the solution is approximate the choice of boundary layer velocity profile is somewhat open. The main physics, however, can be captured with even a crude choice such as

$$u(x, y) = u_\infty \frac{y}{\delta(x)} \quad 0 \leq y \leq \delta, \quad u = u_\infty \quad \delta < y.$$

Use this simple profile. The properties viscosity, μ , and density, ρ , are to be taken as constant.

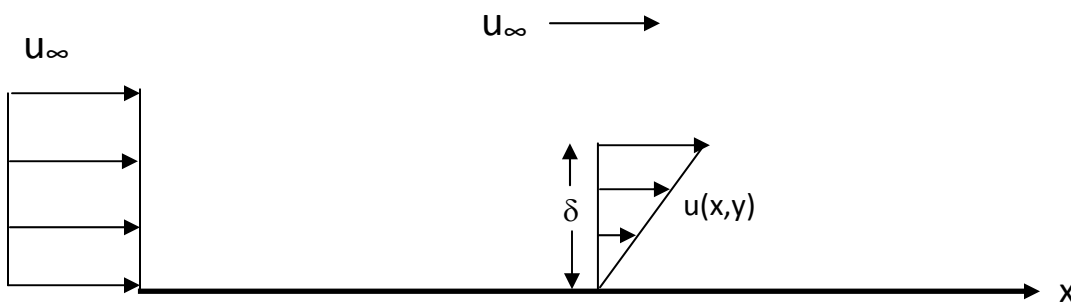
The steady-state control volume equations for x and y momentum are:

$$F_x = F_{Sx} + F_{Bx} = \int_{CS} u \rho \bar{V} \cdot d\bar{A}$$

$$F_y = F_{Sy} + F_{By} = \int_{CS} v \rho \bar{V} \cdot d\bar{A}$$

Here S and B designate surface and body forces. CS is the control surface, while u and v are the x and y components of velocity vector, \bar{V} .

The fluid may be assumed incompressible: $\int_{CS} \bar{V} \cdot d\bar{A} = 0$



Qualifying Exam Spring 2003

Fluid Mechanics

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work **all** three problems.

Problem 1:

The steady, flat plate laminar boundary layer, with zero pressure gradient, can be described by solving the following PDE's. You should recognize them.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

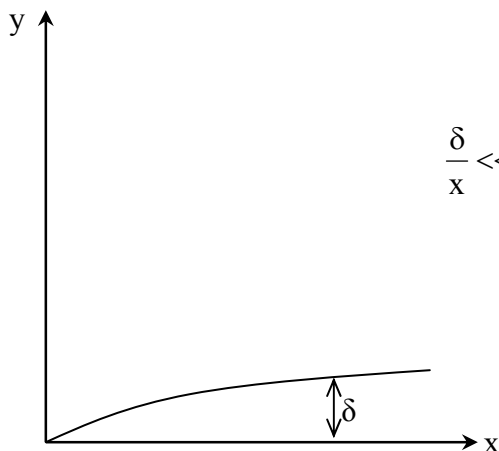
$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

Given the observation that the boundary is very thin when compared to its distance from the leading edge of the plate, derive the simplified boundary layer equations below. You should do this with an order-of-magnitude analysis.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$



Problem 2:

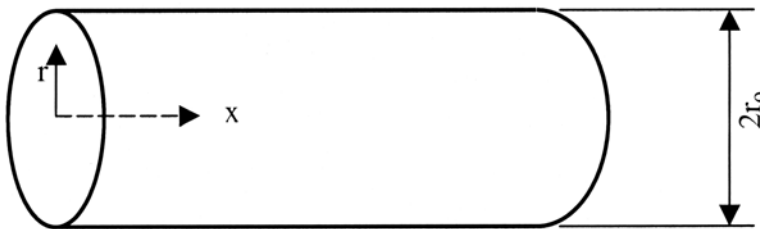
The continuity and momentum equations for 2-D flow for a cylindrical coordinate system are:

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial(rv)}{\partial r} = 0$$

$$u \frac{\partial u}{\partial x} + \frac{v}{r} \frac{\partial(ur)}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right]$$

$$u \frac{\partial v}{\partial x} + \frac{v}{r} \frac{\partial(vr)}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[\frac{\partial^2 v}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) \right]$$

where u and v are velocity components in x and r direction respectively. Simplify the above equations to obtain the momentum equation for hydrodynamically fully developed flow in a circular tube. Use the resulting equation and appropriate boundary condition to obtain velocity distribution for hydrodynamically fully developed flow in a circular tube. Find mean velocity u_m and express the velocity distribution in form of $u/u_m = f(r)$.

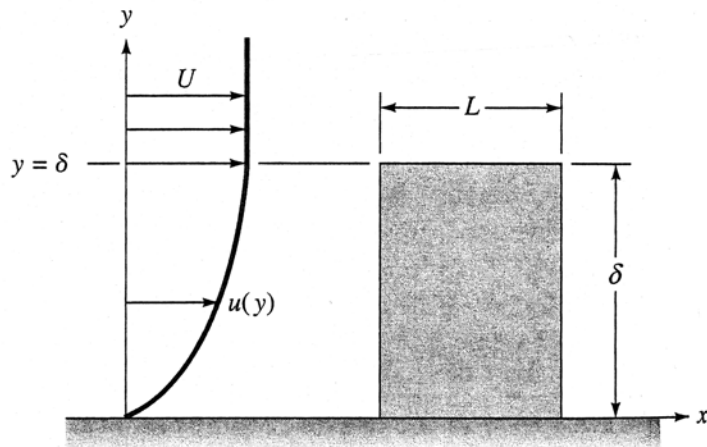


Problem 3:

A flat plate of length L and height δ is placed at a wall and is parallel to an approaching wall boundary layer, as in the figure below. Assume that there is no flow in the y -direction and that in any plane $y = \text{constant}$, the boundary layer that develops over the plate is the Blasius solution for a flat plate. If the approaching wall boundary layer has a velocity profile approximated by:

$$u(y) = U \left[\sin \left(\frac{\pi y}{2\delta} \right) \right]^{2/3}$$

Find an expression for the drag force on the plate. Recall the transformation of variables in the Blasius problem: $\eta = (U_e / 2\nu x)^{1/2} z$ and $u = U_e f'(\eta)$, where U_e is the velocity at the edge of the boundary layer and z is the coordinate normal to the plate. Further, $f''(0) = 0.4696$.



Qualifying Exam Spring 1999

Fluid Mechanics

This portion of the qualifying exam is **closed** book. You may have a calculator.
Work all six problems

Problem 1:

A fully loaded Boeing 777-200 jet transport aircraft weighs 715,000 lbf. The pilot brings the 2 engines to a full takeoff thrust of 102,000 lbf. each before releasing the brakes. Neglecting aerodynamic drag and rolling resistance, estimate the minimum runway length and time needed to reach a takeoff speed of 140 mph. Assume engine thrust remains constant during the ground roll.

Problem 2:

Water, at volume flow rate of $Q = 300$ gpm, is delivered by a fire hose and nozzle assembly. The hose ($L = 200$ ft., $D = 3$ in., and $e/D = 0.004$) is made of four 50 ft. sections joined by couplings. The entrance is squared edged ($K_{en} = 0.5$); the minor loss coefficient for each coupling is $K_c = 0.5$, based upon the mean velocity through the hose. The nozzle loss coefficient is $K_n = 0.02$, based upon the velocity in the exit jet of $D_2 = 1.0$ in. diameter. Estimate the supply pressure required at this flow rate.

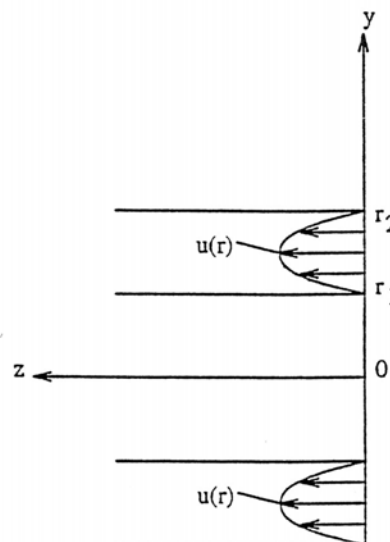
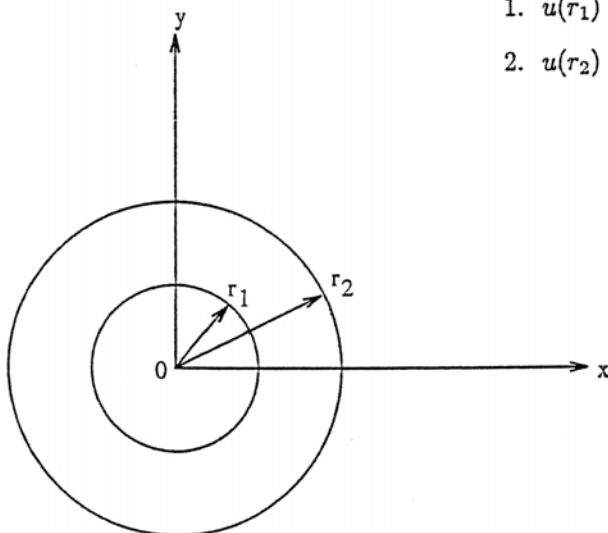
Problem 3:

The differential equation determining the velocity profile for the laminar, incompressible, steady flow of a fluid through a annular conduit is

$$\frac{d^2 w(r)}{dr^2} + \frac{1}{r} \frac{dw(r)}{dr} + \frac{C_o}{\mu} = 0 \quad (1)$$

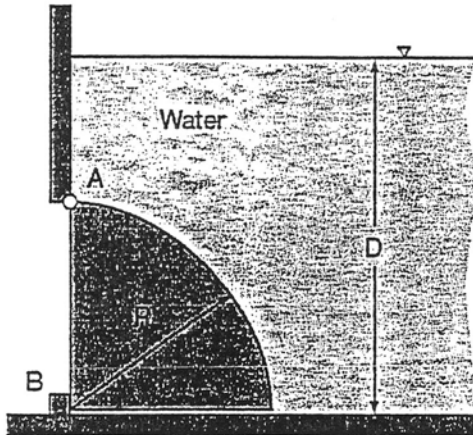
in which $C_o = -\frac{dp}{dz} = \text{const.}$ is the pressure gradient causing the flow. Determine the equation for the velocity profile $u(r)$ by solving Eq.1 if the coefficient of viscosity μ is constant and the boundary conditions are

1. $u(r_1) = 0$
2. $u(r_2) = 0$



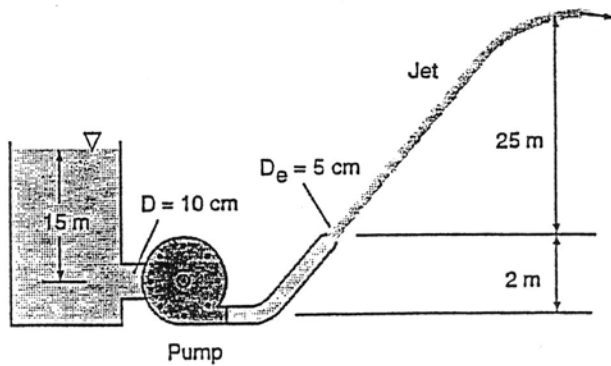
Problem 4:

A gate, in the shape of a quarter-cylinder, hinged at A and sealed at B, is 2 m wide. The bottom of the gate is 3 m below the water surface. Determine the force on the stop at B if the gate is made of concrete (S.G. = 2.4), $R = 2$ m.



Problem 5:

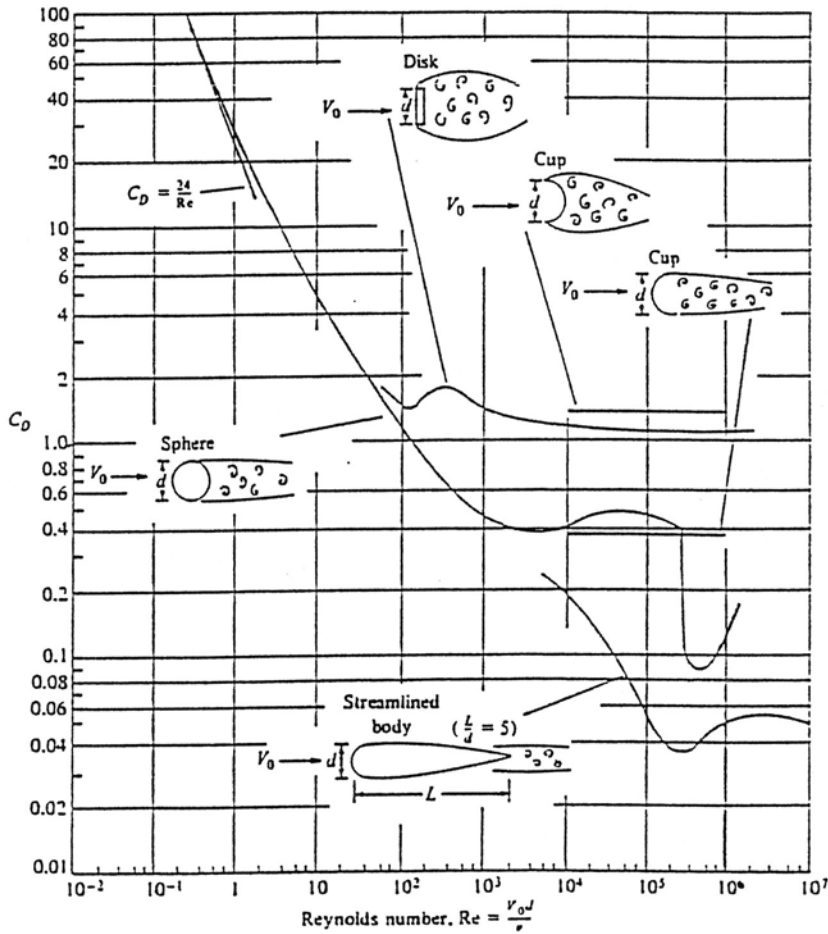
The pump shown creates a water jet oriented to travel a maximum horizontal distance. System friction head losses are 6.5 m. The jet may be approximated by the trajectory of frictionless particles. What power must be delivered by the pump?

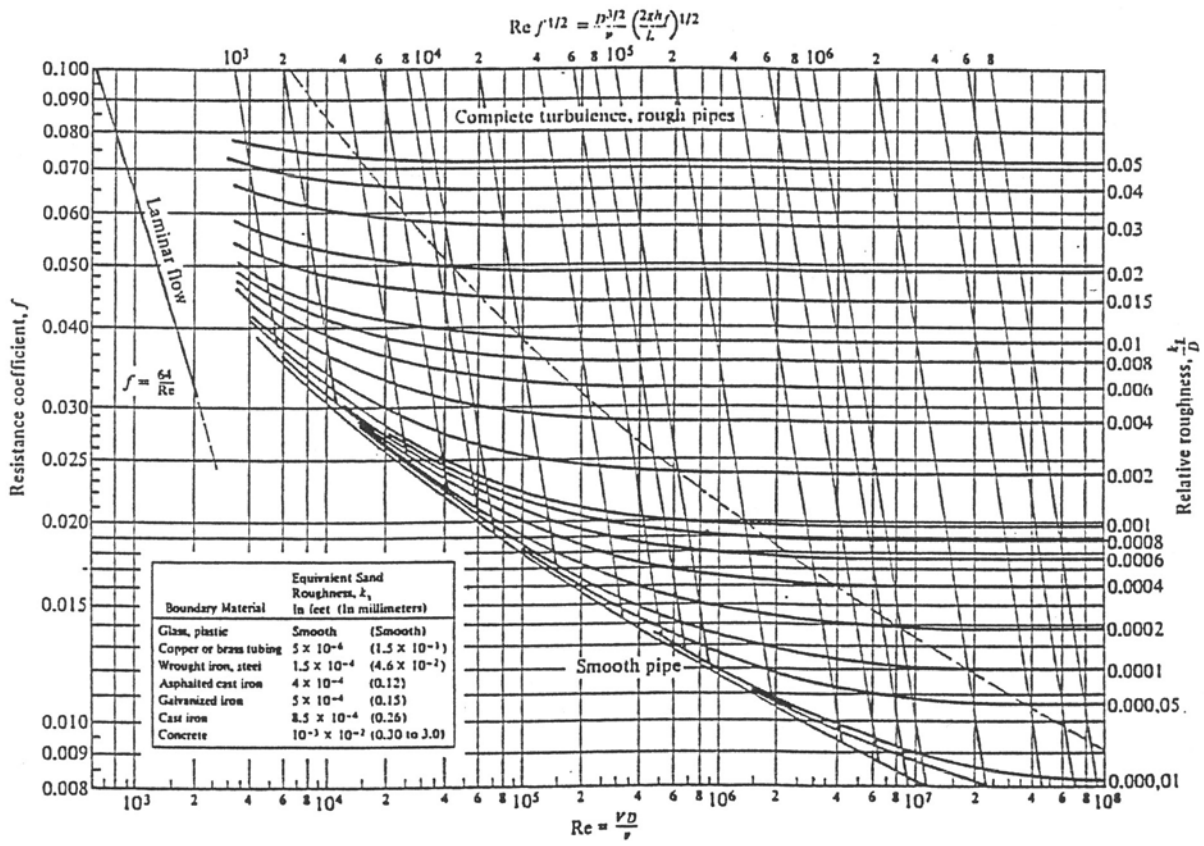


Problem 6:

Consider the graph below, which shows the drag coefficient, C_D , versus the Reynolds number, Re .

- Explain what a boundary layer is and under what circumstances it is encountered.
- Explain the role of the boundary layer in the curve of C_D vs. Re for the sphere shown.
- Explain why the curve of C_D vs. Re is so dramatically different for the sphere and the disk or cup.
- Explain how you could reduce the net drag force on the sphere using boundary-layer control.





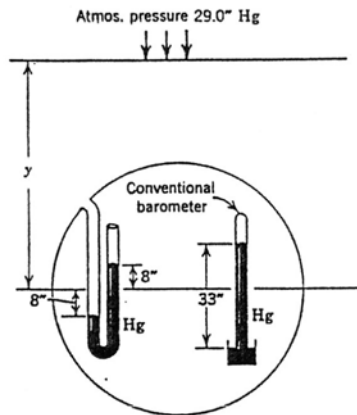
Qualifying Exam Spring 1996

Fluid Mechanics

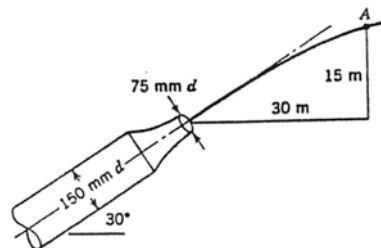
This portion of the qualifying exam is **closed** book. You may have a calculator.

Work all six problems

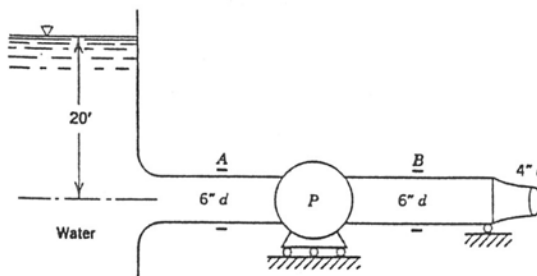
- The sketch shows a sectional view through a submarine. Calculate the depth of submergence, y . Assume the specific weight of sea water to be 64.0 lb/ft^3 .



- The jet passes through a point A, as shown. Calculate the flow rate. Assume the flow is frictionless and incompressible.



- When the pump is started, strain gages at A and B indicate longitudinal tension forces in the pipe of 23 and 100 lb., respectively. Assuming a frictionless system, calculate the flowrate and the pump horsepower.



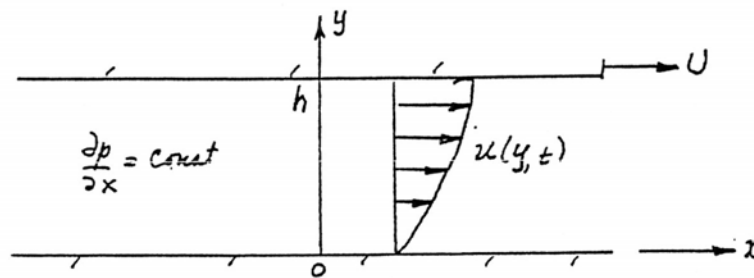
4. When oil (kinematic viscosity $0.001 \text{ ft}^2/\text{sec.}$, specific gravity of 0.92) flows at a mean velocity of 5 fps through a 2-in. pipeline, the head lost in 100 ft of pipe is 18 ft . What will be the headloss when the velocity is increased to 10 fps ?

5. Consider a viscous flow over a wing. What is meant by the boundary layer on the wing?

6. Consider the two-dimensional viscous flow of an incompressible fluid through a conduit whose upper wall is moving with a constant known velocity, U , as shown below. For this so-called Couette flow the Navier-Stokes equation reduces to

$$\frac{\partial u(y, t)}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u(y, t)}{\partial y^2}$$

Assume that $\partial p / \partial x = \text{const.} = \rho C_0$, that the kinematic viscosity ν is constant and solve this equation for the velocity $u(y, t)$. Assume the initial condition is $u(y, 0) = 0$.



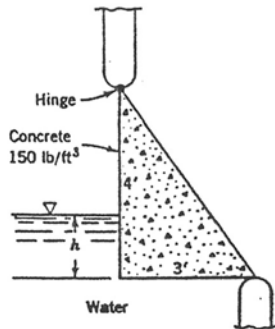
Qualifying Exam Spring 1995

Fluid Mechanics

This portion of the qualifying exam is **closed** book. You may have a calculator.

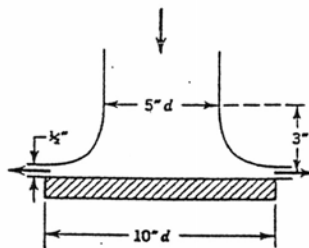
Work all six problems

1. Calculate the h at which the gate will open.

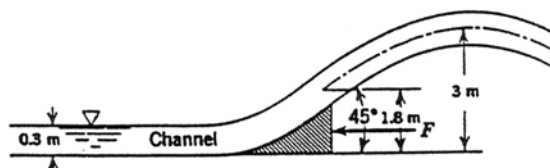


2. A manifold pipe of 3 inch diameter has four openings in its walls spaced equally along the pipe and is closed at its downstream end. If the discharge from each opening is $0.5 \text{ ft}^3/\text{sec}$, what are the mean velocities in the pipe between the openings? Assume a frictionless, incompressible flow.

3. If a free jet of fluid strikes a circular disk and produces the flow pattern shown, what is the flow rate? Assume an incompressible, ideal fluid.



4. Calculate the force, F , required to drive the scoop (hatched) at such a velocity that the top of the jet centerline is 3m above the channel bottom. Assume the scoop and channel are 1.5m wide normal to the paper and that the scoop extracts all the water from the channel. Assume frictionless, incompressible flow.



5. When a liquid flows in a horizontal pipe which has a diameter of 150mm, the shear stress at the wall is $100N/m^2$. Calculate the the pressure drop in 30m of this pipeline. What is the shearing stress in the liquid 25mm from the pipe centerline? Assume an incompressible, steady flow.

6. (a.) What is a boundary-layer? (b.) Discuss Prandtl's boundary-layer approximations to the Navier-Stokes equations. (c.) Blasius' non-dimensional form of the boundary-layer equation for steady, incompressible flow past a flat plate is

$$\frac{d^3 f(\eta)}{d\eta^3} + \frac{1}{2}f(\eta)\frac{d^2 f(\eta)}{d\eta^2} = 0 \quad ,$$

in which $f(\eta)$ is the non-dimensional stream function and η is the non-dimensional coordinate normal to the plate. What are the boundary conditions for this equation? Discuss what method(s) you might use to solve it. Note the dimensional velocity in the direction parallel to the plate in the boundary-layer is

$$u = U \frac{df(\eta)}{d\eta}$$

where U is the constant uniform flow parallel to the flat plate outside the boundary-layer.

Qualifying Exam Spring 1992

Fluid Mechanics

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work all problems

Problem 1. Consider the viscous flow found in a journal bearing as shown. The top block is fixed, the bottom plate moves from left to right with constant velocity U .

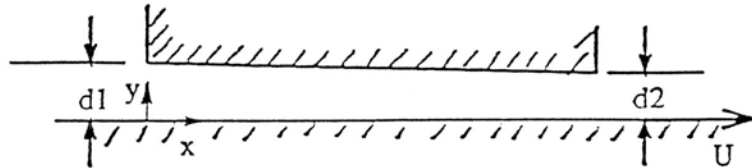
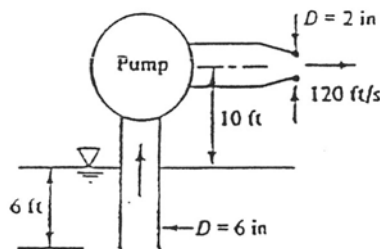


Figure 2: Journal Bearing

- a.) Develop the equation of motion from basic principles, using a free-body diagram of a fluid element. Your result should be in Eulerian form, $u(x, y, z, t)$.
- b.) Simplify the equation of motion for steady conditions, and considering the bearing gap, d , to vary only very slightly with x .
- c.) Using a solution of the above for constant gap height d , find $u(y)$. Use this to find the flow rate through the bearing per unit depth as a function of d and the pressure gradient.
- d.) Now considering d to vary linearly with x , find the force components on the upper block, and finally the ratio of tangential to normal force. You may consider the ambient pressure at each end of the journal to be the same. Comment on your result and your approach.

Problem 2. A fireboat pump draws seawater ($SG=1.025$) from a 6-in. submerged pipe and discharges it at 120 ft/sec through a 2-in. nozzle, as shown. Total head loss is 8 ft. If the pump is 70% efficient, how much horsepower is required to drive it? Assume uniform, steady flow.



Qualifying Exam Spring 1991

Fluid Mechanics

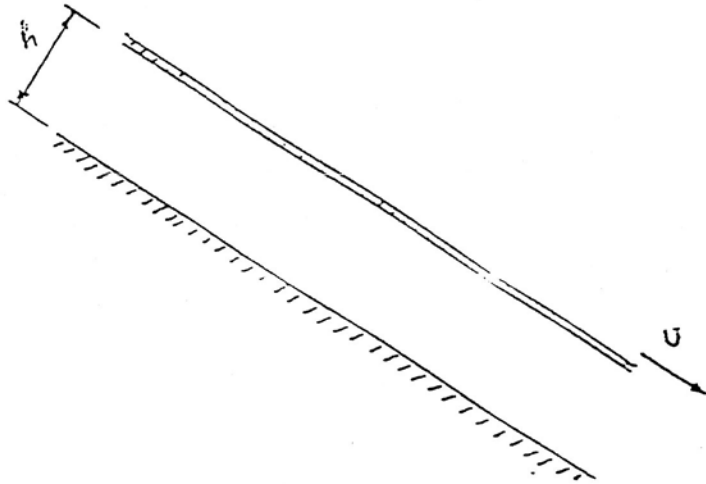
This portion of the qualifying exam is **closed** book. You may have a calculator.

Work all problems

Problem 1:

Consider the the flow between the parallel plates as shown. The top plate is moving with velocity U . Assuming the flow to be fully-developed and laminar,

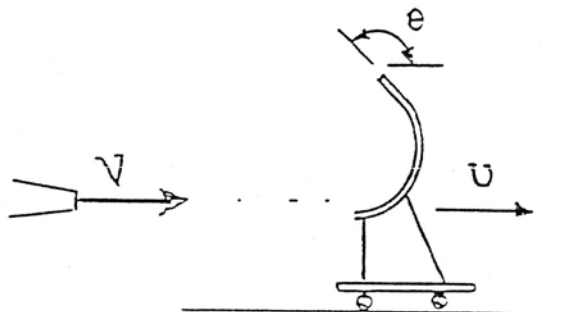
- find the equation of motion,
- find an expression for the shear stress at the bottom surface,
- find the velocity distribution and draw what you expect to be a typical velocity profile.



Problem 2:

Consider a single vane with turning angle, θ , moving horizontally at constant speed, U . A jet of fluid with absolute velocity, V , strikes the vane.

- Find the force which is delivered to the vane.
- Find the power which the vane could deliver under the action of the water jet.
- Find the value of U/V to maximize the power delivered by the jet.



Qualifying Exam Spring 1990

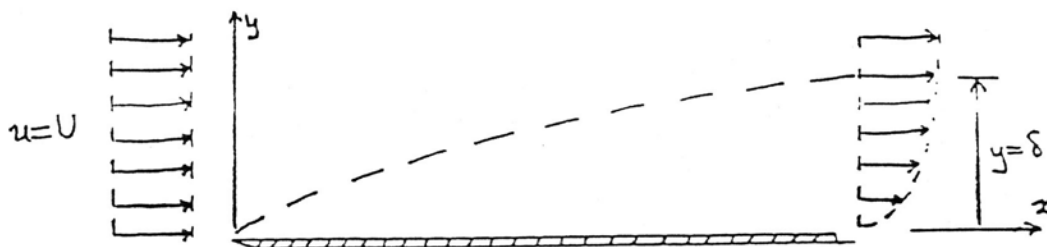
Fluid Mechanics

This portion of the qualifying exam is **open** book. You may have a calculator.

Work all problems

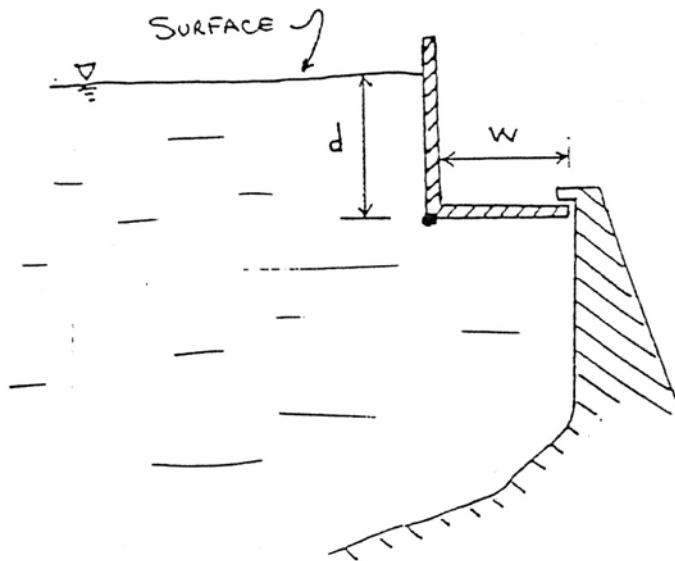
Question 1.

A uniform flow with velocity $u = U$ passes over a flat plate which is parallel with the flow to direction. A boundary layer develops near the surface of the plate as shown. Show that the drag force (per unit depth) of the fluid on the surface is given by $D = \int_0^\delta \rho u (U - u) dy$ where δ is the thickness of the boundary layer.



Question 2.

A water gate in the shape of an L is shown. The gate maintains this L shape, but is free to rotate about the hinge as shown. As water rises on the left side of the gate, the gate will automatically open. Find the depth d above the hinge at which this will occur. Neglect the mass of the gate.

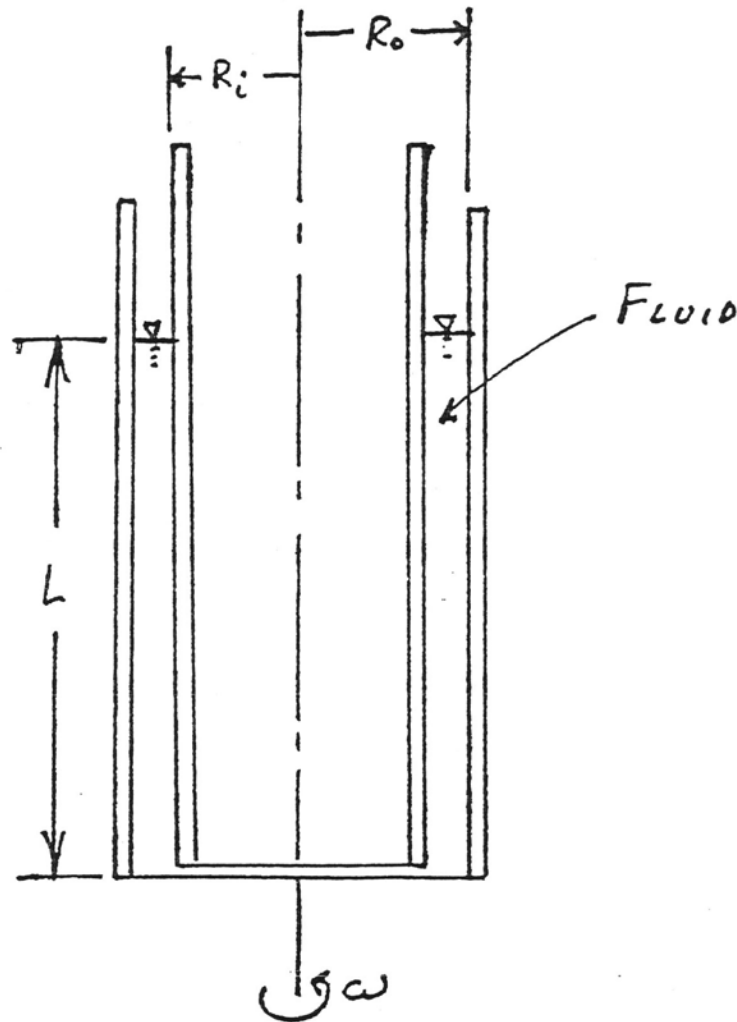


Question 3.

What is the fluid dynamic reason for dimpling a golf ball?

Question 4.

Two concentric cylinders are used to make a viscometer. The inner cylinder is stationary while the outer cylinder rotates at an angular velocity ω . Assume that the fluid is Newtonian and incompressible with constant properties. Assuming that the gap is small compared to the radius, derive an equation for the viscosity μ in terms of the torque required to rotate the cylinder at the steady velocity ω . The cylinder radii are R_i and R_o , and the fluid is in contact with a cylinder of length equal to L . Neglect any end effects and any friction due to bearings or seals. The flow is laminar with no secondary motion.



Qualifying Exam Spring 1988

Fluid Mechanics

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work all problems

1. Derive an expression for the velocity field around an infinite circular cylinder for an incompressible, irrotational flow given that the velocity field is uniform at an infinite distance away from the cylinder.
2. If the flow is viscous, discuss how the velocity field would differ from the inviscid case.
3. Discuss each of the following:
 - a.) What is an inviscid flow?
 - b.) What is an irrotational flow? Can a viscous flow be irrotational? Can an inviscid flow be rotational?
 - c.) What is a streamline, streakline, and a pathline?
 - d.) The velocity field for an irrotational, incompressible flow can be written in terms of a scalar potential. Give the mathematical expression for this relationship.
 - e.) How are the streamlines oriented relative to the equipotential lines for an irrotational incompressible flow?
4. The equation modelling the transport of some quantity, ϕ , in a transient one-dimensional convective system with a uniform velocity is given by

$$\frac{\partial \phi}{\partial t} + V \frac{\partial \phi}{\partial x} = 0$$

where V is the fluid velocity, x is position, and t is time. Approximate the above equation with a finite difference equation. Keep it simple. Discuss the problems associated with modelling the above equation with finite differences. Discuss numerical dissipation, dispersion, and Gibb's error.

5. An approximation for the velocity profile of a 2-D laminar boundary layer is

$$\frac{u}{U} = \frac{3}{2} \left(\frac{y}{\delta} \right) - \frac{1}{2} \left(\frac{y}{\delta} \right)^3$$

where δ is the local boundary layer thickness, such that

$$\frac{\delta}{x} = \frac{4.64}{\sqrt{R_{e_x}}}$$

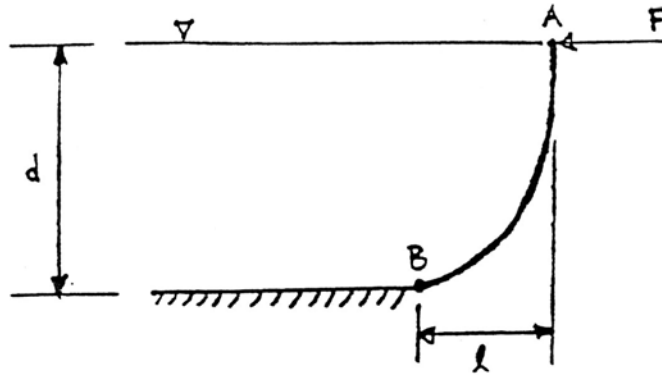
$$R_{e_x} = \frac{xU}{\nu} = \text{Reynold's Number}$$

and U is the free stream velocity. If the fluid can be considered incompressible, find an expression for the y velocity component v .

Qualifying Exam Miscellaneous Fluid Mechanics

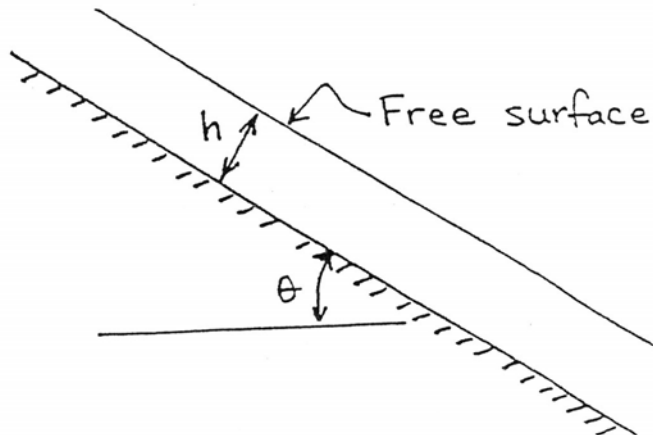
Gate AB has width W into the paper, is parabolic in shape, and is hinged at B . The open-surface tank is filled to depth d with water, $\rho = 998 \text{ kg/m}^3$. Given $W = 10 \text{ m}$, $d = 8 \text{ m}$, and $l = 5 \text{ m}$;

- a) Find the resultant horizontal and vertical hydrostatic forces on the gate.
- b) Find the lines of action of these forces.
- c) What is the magnitude of force F required to keep the gate closed?



Consider the the fully-developed laminar flow down the inclined surface shown below.

- a) Draw a free-body diagram for a fluid element.
- b) Draw what you expect to be a typical velocity profile and state the boundary conditions.
- c) Find an expression for the shear stress at the surface.
- d) Find an expression for the flow rate per unit depth down the incline.



Name _____

Fall 2006

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #___, #___, and #___ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

3. Consider a beaker of water in which are contained a few small bits of tea leaves that have absorbed water and sunk to the bottom. The fact that they are tea leaves is not important, rather what is important is that there are some small bits of matter that are somewhat denser than water.

Now a spoon or swizzle stick is use to vigorously stir the water in a circular fashion, causing the water to rotate more or less about the vertical axis of the beaker centerline. At first the tea leaves are dispersed, but are then observed to sink back to the bottom and migrate toward the center of the beaker bottom where they remain.

Please explain this behavior form a fluid mechanical standpoint.

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

4. You are to use an integral control volume analysis to determine the laminar boundary layer thickness δ as a function of x . Consider the flow over a smooth flat plate of a Newtonian fluid, with no pressure gradient in the flow direction. As the solution is approximate the choice of boundary layer velocity profile is somewhat open. The main physics, however, can be captured with even a crude choice such as

$$u(x, y) = u_\infty \frac{y}{\delta(x)} \quad 0 \leq y \leq \delta, \quad u = u_\infty \quad \delta < y.$$

Use this simple profile. The properties viscosity, μ , and density, ρ , are to be taken as constant.

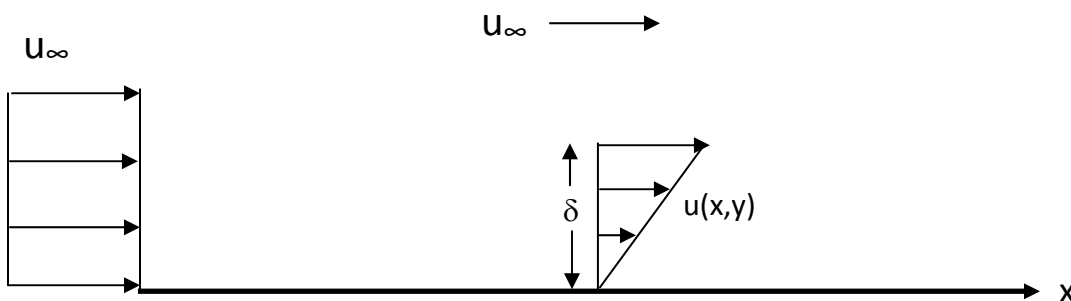
The steady-state control volume equations for x and y momentum are:

$$F_x = F_{Sx} + F_{Bx} = \int_{CS} u \rho \bar{V} \cdot d\bar{A}$$

$$F_y = F_{Sy} + F_{By} = \int_{CS} v \rho \bar{V} \cdot d\bar{A}$$

Here S and B designate surface and body forces. CS is the control surface, while u and v are the x and y components of velocity vector, \bar{V} .

The fluid may be assumed incompressible: $\int_{CS} \bar{V} \cdot d\bar{A} = 0$



Name _____

Spring 2007

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

This portion of the qualifying exam is **closed** book. You may have a calculator.

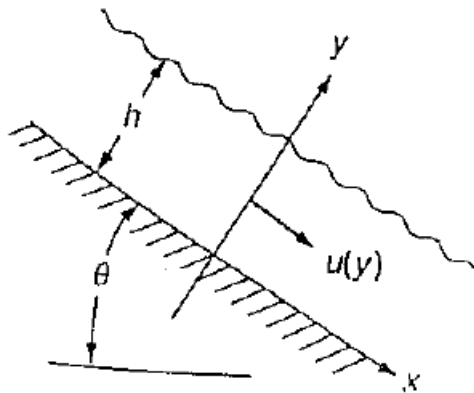
Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

1. Consider the incompressible flow of a fluid of viscosity μ down an inclined plane, as shown in the figure below. Assume that the flow is steady, one-dimensional (i.e. the only non-zero component of velocity is along the x-axis) and the atmosphere exerts constant pressure and negligible shear on the free surface. Derive an expression for $u(y)$. (Note: the figure is a cartoon, ignore the 'waves' you see on the surface).

FIGURE 1

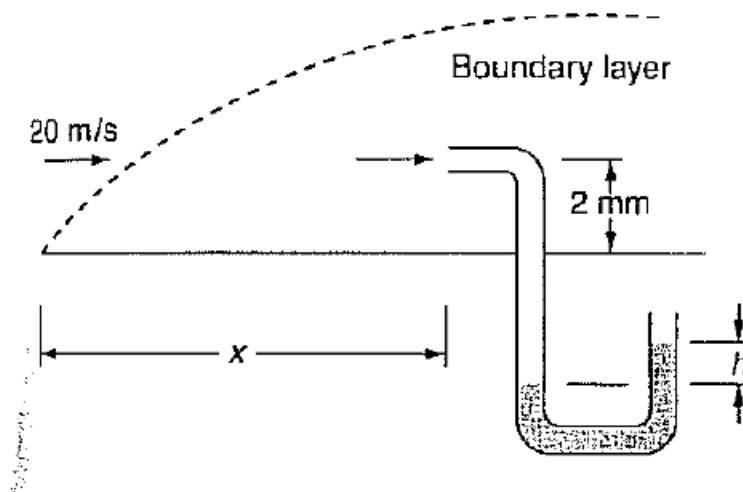


Qualifying Exam: Fluid Mechanics

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2. Air at standard conditions flows past a smooth flat plate, as in the Figure below. A pitot stagnation tube, placed 2 mm from the wall, develops a water manometer head $h = 21$ mm.
- Estimate the flow speed parallel to the plate at the location of the tube.
 - Assuming a laminar flat plate boundary layer, estimate the position x of the tube.

FIGURE 2



1. Navi

$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i, \quad i = 1, 2, 3$$

2. Continuity equation for Incompressible Flows

$$\frac{\partial u_i}{\partial x_i} = 0$$

3. Bernoulli's Equation

$$p + \frac{1}{2} \rho V^2 + \rho g z = \text{constant along a streamline}$$

4. Constitutive relation for Newtonian fluids

$$\tau_{ij} = -p \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

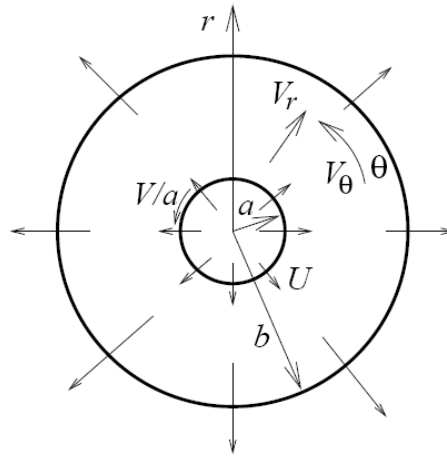
Qualifying Exam: Fluid Mechanics

CLOSED BOOK

3.

Consider steady incompressible flow between two concentric cylinders shown below, with radii a and b , respectively. Liquid of constant kinematic viscosity ν fills the gap between the cylinders. The inner cylinder (radius a) rotates at a constant angular rate (V/a) and the outer cylinder (radius b) is fixed in space.

Liquid seeps through the inner cylinder radially at a constant velocity U ; fluid also seeps radially through the outer cylinder. ASSUME that flow parameters only depend on the radial distance r measured from the center of cylinder a .



(a) Show that the radial velocity component in the entire gap region is given by

$$v_r = \frac{aU}{r}$$

(b) Write down the governing equation for v_θ , the tangential velocity component in the gap, and derive the solution for $v_\theta(r)$ with the given boundary conditions.

(hint: 2D steady incompressible N-S equations in cylindrical coordinate

$$\frac{1}{r} \frac{\partial}{\partial r}(rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0$$

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} = \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r}(rv_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} \right] - \frac{\partial p}{\partial r}$$

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} = \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r}(rv_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right] - \frac{1}{r} \frac{\partial p}{\partial \theta}$$

)

Name _____

Spring 2007

Qualifying Exam: Fluid Mechanics

CLOSED BOOK

4. Please discuss the various contributions to fluid dynamical drag, paying particular attention to the mechanisms and their relative contribution to total drag for the following situations.
 - a. Fully immersed object with Reynolds number less than one.
 - b. Fully immersed object with Reynolds number much greater than one.
 - c. Object moving at fluid interface such as a ship on the ocean.