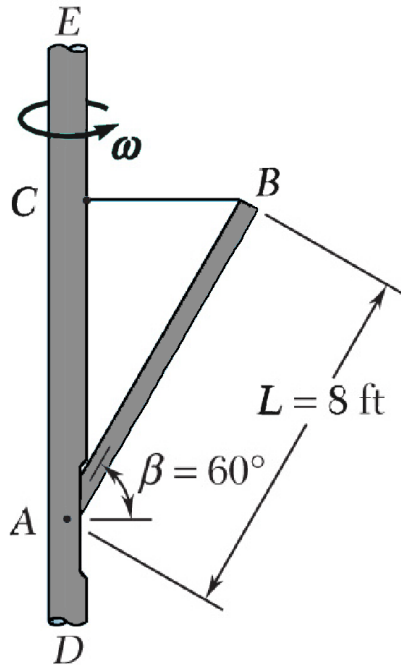


## Qualifying Exam: Dynamics and Vibrations

## CLOSED BOOK

1. Rod  $AB$  with weight  $W = 40$  lb is pinned at  $A$  to a vertical axle which rotates with constant angular velocity  $\omega = 15$  rad/s. The rod position is maintained by a horizontal wire  $BC$ . Determine the tension in the wire and the reaction at  $A$ .



Name: \_\_\_\_\_

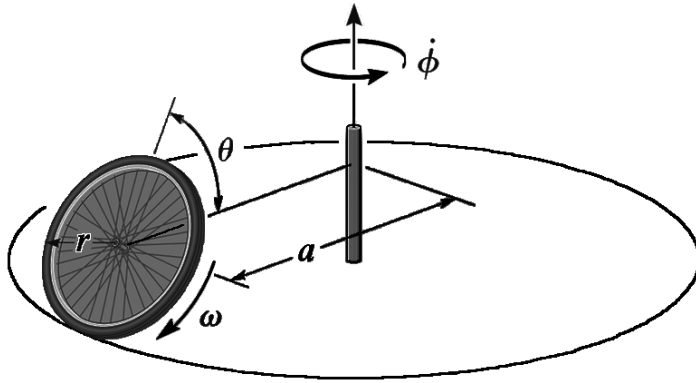
Fall 2014

**Qualifying Exam: Dynamics and Vibrations**

**CLOSED BOOK**

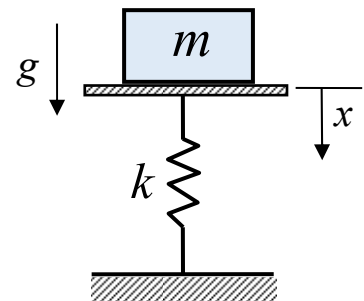
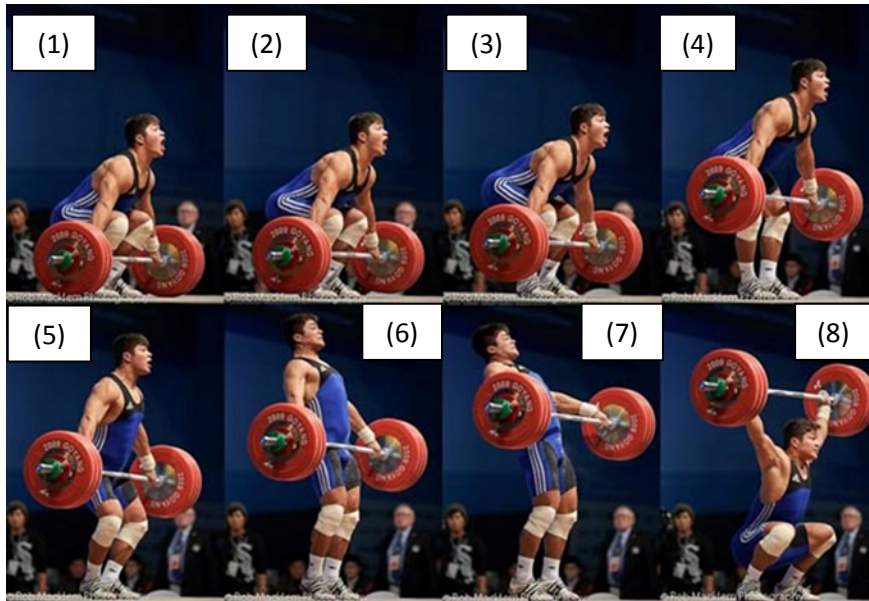
---

2. A wheel of mass  $m$  and radius  $r$  rolls with constant spin  $\omega$  about a circular path having a radius  $a$ . If the angle of inclination is  $\theta$ , determine the rate of precession. Treat the wheel as a thin ring. No slipping occurs.



Problem 1. A male athlete is trying to lift a weight, as depicted in the following. By modeling a human body as a spring, explain that the actual weight the athlete may experience at the very beginning of the stage 8 could be two times of the weight that he is lifting. Note that a forced response of a mass-spring system,  $m\ddot{x} + kx = F(t)$ , can be computed, in the form of convolution integral, as

$$x(t) = \int_0^t F(\tau)h(t-\tau)d\tau = \int_0^t F(t-\tau)h(\tau)d\tau \text{ where } h(t) = (\sin \omega_n t) / (m\omega_n) \text{ and } \omega_n = \sqrt{k/m}.$$



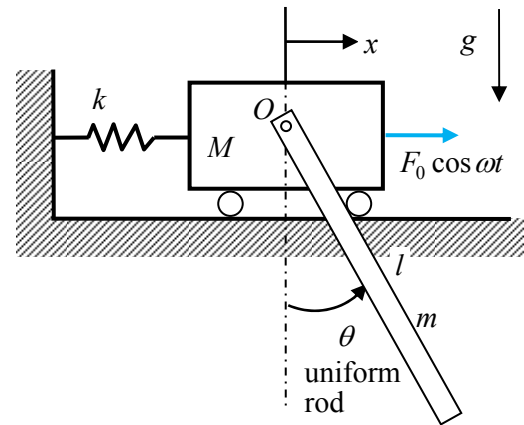
$x=0$  when the spring is unstretched or uncompressed.

At  $t=0$ ,  $x=0$  with zero velocity.

Problem 2. Assuming that the system is initially at rest and that the motions are small, find the length of the pendulum that renders the mass  $M$  to be stationary under this harmonic excitation (i.e.,  $x(t) = 0$ ). Also, calculate the corresponding steady-state amplitude of the pendulum. Note that, since the mass moment of inertia of the uniform rod about its cg is  $ml^2 / 12$ , the mass moment of inertia of the rod about the hinge point  $O$  is  $ml^2 / 3$  by the parallel axis theorem. First, show the linearized equations of motion can be written as

$$(M + m)\ddot{x} + (ml / 2)\ddot{\theta} + kx = F_0 \cos \omega t$$

$$(ml^2 / 3)\ddot{\theta} + (ml / 2)\ddot{x} + (mgl / 2)\theta = 0$$



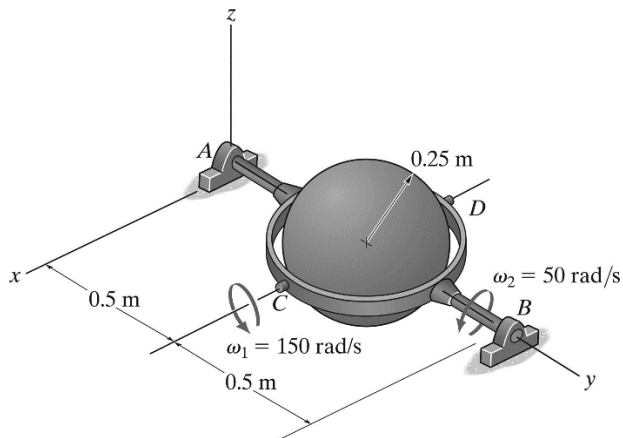
Name: \_\_\_\_\_

Spring 2014

### Qualifying Exam: Dynamics and Vibrations

#### CLOSED BOOK

1. The 20-kg sphere is rotating with a constant angular speed of  $\omega_1 = 150$  rad/s about axle  $CD$ , which is mounted on the circular ring. The ring rotates about shaft  $AB$  with a constant angular speed of  $\omega_2 = 50$  rad/s. If shaft  $AB$  is supported by a thrust bearing at  $A$  and a journal bearing at  $B$ , determine the  $X, Y, Z$  components of reaction at these bearings at the instant shown. Neglect the mass of the ring and shaft.



Name: \_\_\_\_\_

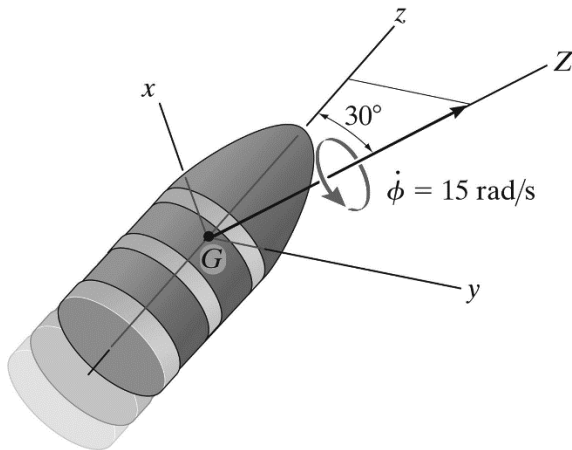
Spring 2014

**Qualifying Exam: Dynamics and Vibrations**

**CLOSED BOOK**

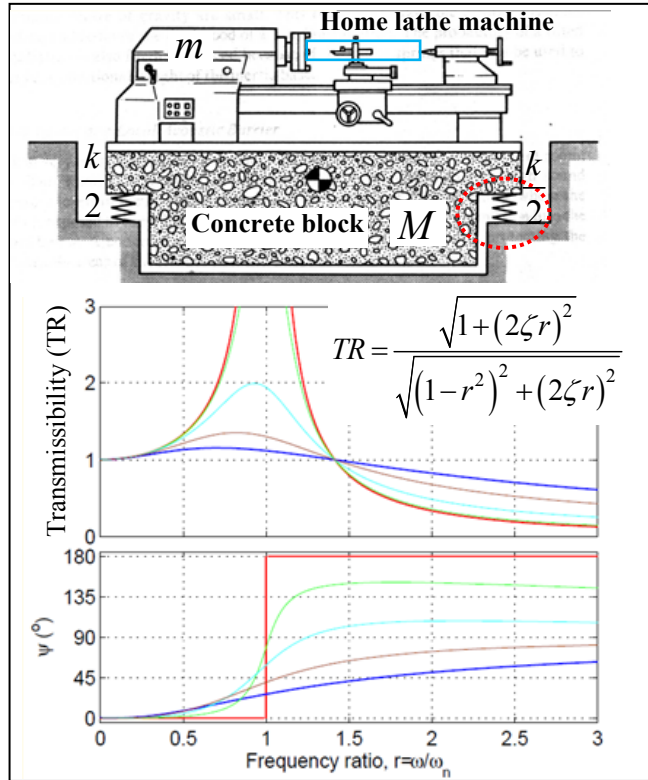
---

2. The projectile precesses about the  $Z$  axis at a constant rate of  $\dot{\phi} = 15$  rad/s when it leaves the barrel of a gun. Determine its spin  $\dot{\psi}$  and the magnitude of its angular momentum  $\mathbf{H}_G$ . The projectile has a mass of 1.5 kg and radii of gyration about its axis of symmetry ( $z$  axis) and about its transverse axes ( $x$  and  $y$  axes) of  $k_z = 65$  mm and  $k_x = k_y = 125$  mm, respectively.



Problem 1. What would be the maximum weight reading if 3kg mass is dropped off instantly on the top of the spring scale? Model the system as a mass-spring system with zero initial conditions, including the gravitational effect. Note that a forced response of a mass-spring system,  $m\ddot{x} + kx = F(t)$ , can be computed, in the form of convolution integral, as  $x(t) = \int_0^t F(\tau)h(t-\tau)d\tau = \int_0^t F(t-\tau)h(\tau)d\tau$  where  $h(t) = \frac{1}{m\omega_n} \sin \omega_n t$  and  $\omega_n = \sqrt{k/m}$ .

Problem 2. When a home lathe machine (which weighs 1.2 kN) was purchased, it was directly mounted on four vibration isolators (whose total stiffness becomes 270 kN/m) to the ground. Then, a disturbance of 250 N that the machine experiences during the normal operation speed at a certain rpm due to rotating unbalance and cutting process was reduced to 50 N on the ground (i.e., 80% reduction of disturbance). Now, to reduce vibration amplitudes in the lathe machine at the normal operation speed, a concrete block is added beneath the lathe machine and the whole system is mounted on twelve isolators. What is the normal operation speed of the lathe machine? What is the appropriate weight of the concrete block to maintain the same transmissibility, and how much the vibration amplitude in the lathe machine will be reduced? Neglect damping and assume the vertical vibrations only.

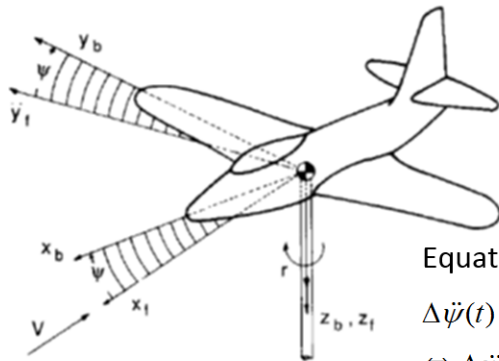




**Qualifying Exam: Dynamics and Vibrations**

**CLOSED BOOK**

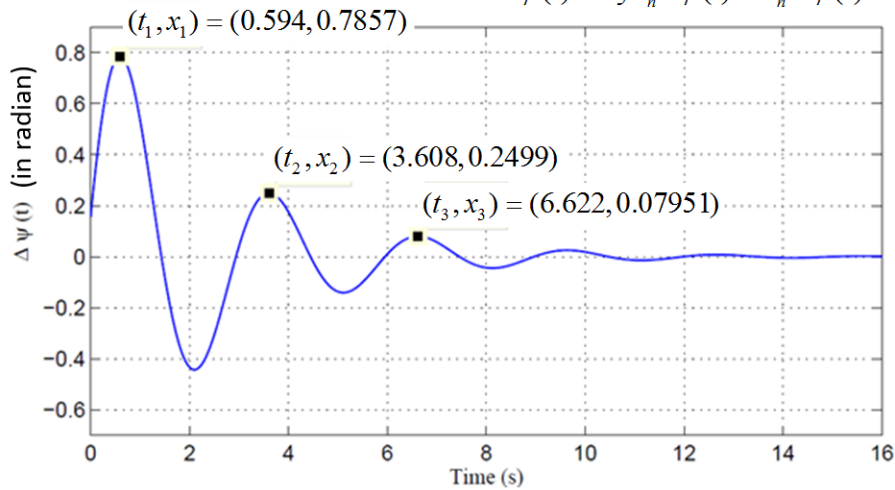
1. A pure yawing motion test of an airplane was performed to estimate stability derivatives associated with lateral-directional flight stability of the airplane. When the perturbed yawing angle,  $\Delta\psi(t)$ , was measured, what are these two stability derivatives,  $N_r$  and  $N_\beta$  ?



Equation of motion:

$$\Delta\ddot{\psi}(t) - N_r\Delta\dot{\psi}(t) + N_\beta\Delta\psi(t) = 0$$

$$\Leftrightarrow \Delta\ddot{\psi}(t) + 2\zeta\omega_n\Delta\dot{\psi}(t) + \omega_n^2\Delta\psi(t) = 0$$



## Qualifying Exam: Dynamics and Vibrations

### CLOSED BOOK

2. Figure 1 shows an *air spindle* system. The system consists of a circular turntable that can rotate freely about the vertical (out-of-plane) axis passing through its center, and two proof masses mounted on slots that are offset from the center of the turntable. We make the following assumptions:
1. The platform is perfectly leveled so that gravity has no influence on the system;
  2. The external disturbances, e.g. friction and aerodynamics, can be ignored;
  3. The proof masses can be modeled as mass points.

We introduce the following notation with reference to Figure 1:

- $I$  = the inertia of the platform including components mounted on the platform;
- $\theta$  = the attitude angle of the platform;
- $m_i$  = the mass of the  $i$ -th proof mass for  $i = 1, 2$ ;
- $z_i$  = the relative position of the  $i$ -th proof mass with respect to a platform-fixed rotating frame,  $i = 1, 2$ ;
- $l_i$  = the normal distance from the center of mass of the platform to the  $i$ -th slot,  $i = 1, 2$ .

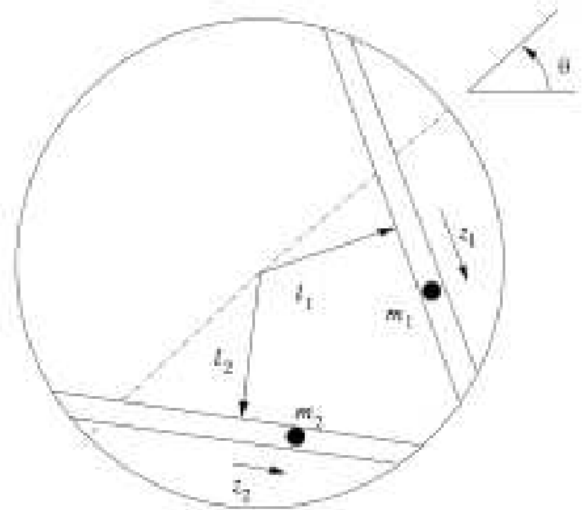


Figure 1: *Air spindle system with two proof masses on their respective slots, offset from the center of rotation of the turntable.*

We choose  $z_i = 0$  to correspond to the position in the  $i$ -th slot whose distance to the rotation axis of the platform is minimum; this minimum distance is  $l_i$ .

- a) Find the equations of motion of this system using the Euler-Lagrange equations.
- b) Find the total angular momentum of the air spindle system and show that it is conserved.
- c) Give the relation between the angular rate  $\dot{\theta}$ , and position rates  $\dot{z}_1$  and  $\dot{z}_2$  when the total angular momentum is zero.

Name: \_\_\_\_\_

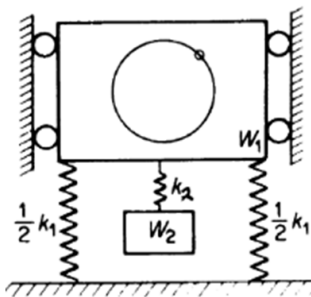
Fall 2013

**Qualifying Exam: Dynamics and Vibrations**

**CLOSED BOOK**

---

3. A mathematical model is considered for reducing an engine vibration of an automobile with a primary system (the engine) of weight 200 lb and the absorber of 50 lb. If the primary mass is excited by a 2 lb-in unbalancing rotating at 1800 r.p.m., determine the proper value of the absorber spring  $k_2$  to eliminate the unbalanced perturbation in the primary system. What will be the amplitude of the absorber?



Name: \_\_\_\_\_

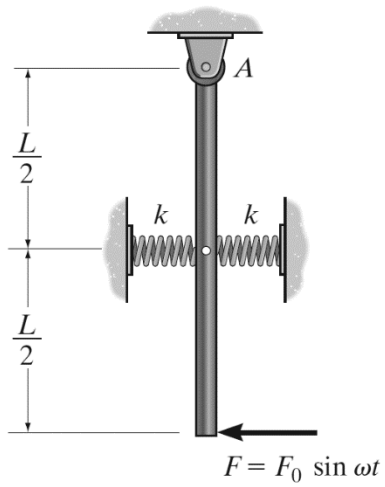
Fall 2013

**Qualifying Exam: Dynamics and Vibrations**

**CLOSED BOOK**

---

4. The uniform rod has a mass of  $m$ . If it is acted upon by a periodic force of  $F = F_0 \sin \omega t$ , determine the amplitude of the steady-state vibration.



Name \_\_\_\_\_

**Qualifying Exam: Dynamics & Vibrations**

**CLOSED BOOK**

<b>Path Variables:</b>	$\bar{v} = v\hat{e}_t$	$\bar{a} = \dot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$
<b>Rectangular Coordinates Constant Acceleration:</b>	$v = v_0 + at$	$s = s_0 + v_0t + \frac{1}{2}at^2$ $v^2 - v_0^2 = 2(s - s_0)$
<b>Cylindrical Coordinates:</b>	$\bar{r} = r\hat{e}_r + z\hat{e}_z$ $\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta + \ddot{z}\hat{e}_z$	$\bar{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{e}_z$
<b>2-D Equations of Motion:</b>	$\sum \bar{F} = m\bar{a}_G$	$\sum \bar{M}_G = I_G\bar{\alpha}$ $\sum (\bar{r}_i \times \bar{F}_i) + \sum \bar{M}_i = \bar{r}_G \times m\bar{a}_G + I_G\bar{\alpha}$
<b>3-D Equations of Motion:</b>	$\sum \bar{F} = m\bar{a}_G$ $\sum \bar{M}_A = \dot{\bar{H}}_A$ $\bar{\omega} = \bar{\Omega}$ $\sum M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$ $\sum M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$ $\sum M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y$	$\dot{\bar{H}}_A = (\dot{\bar{H}}_A)_{xyz} + \bar{\omega} \times \bar{H}_A$ $(\dot{\bar{H}}_A)_{xyz} = (I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z)\hat{i} + (I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z)\hat{j} + (I_{zz}\alpha_z - I_{xz}\alpha_x - I_{yz}\alpha_y)\hat{k}$ $\bar{\omega} \neq \bar{\Omega}$ $\sum M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$ $\sum M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x$ $\sum M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y$
<b>Work – Energy:</b>	$\Delta T + \Delta V = (T_2 - T_1) + (V_2 - V_1) = U_{1 \rightarrow 2}^{(nc)}$ $T = \frac{1}{2} m(\bar{v}_G \cdot \bar{v}_G) + \frac{1}{2} I_G\omega^2$	$U_{1 \rightarrow 2} = \sum \int_{r_1}^{r_2} (\bar{F} \cdot d\bar{r}) = \sum \int_{\theta_1}^{\theta_2} (\bar{M} \cdot d\bar{\theta})$ $V_{gravity} = mgy$ $V_{spring} = \frac{1}{2} k\delta^2$
<b>Power:</b>	$P = \bar{F} \cdot \bar{v}$	$\varepsilon = \text{efficiency} = \frac{\text{power output}}{\text{power input}}$
<b>Impulse – Momentum:</b>	$\sum \int \bar{F} dt = \Delta m\bar{v}_G = (m\bar{v}_G)_2 - (m\bar{v}_G)_1$ $e = \text{coef of restitution} = \frac{(v_B)_2 - (v_A)_1}{(v_A)_1 - (v_B)_2}$	$\sum \int \bar{M}_A dt = \Delta \bar{H}_A = (\bar{H}_A)_2 - (\bar{H}_A)_1$ $\bar{H}_G = I_G\bar{\omega}$ $\bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$

Name \_\_\_\_\_

**Qualifying Exam: Dynamics & Vibrations**

**CLOSED BOOK**

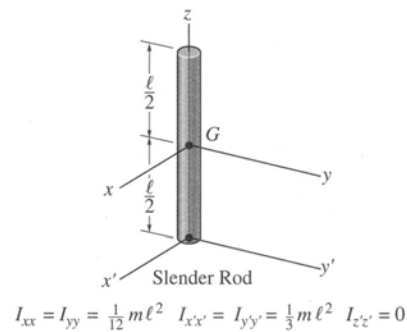
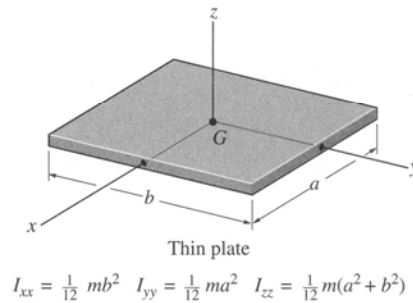
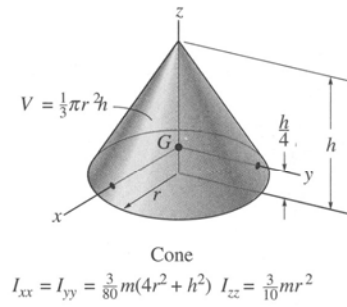
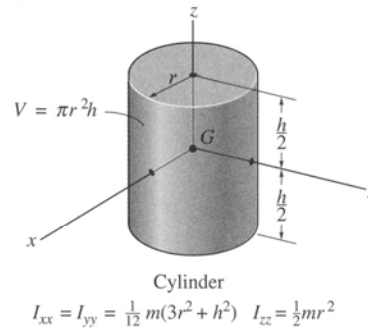
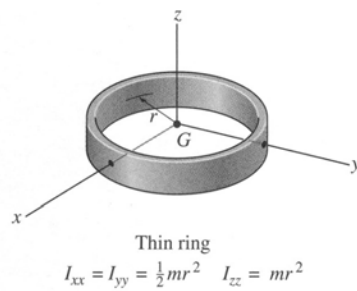
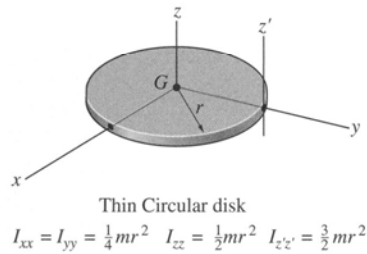
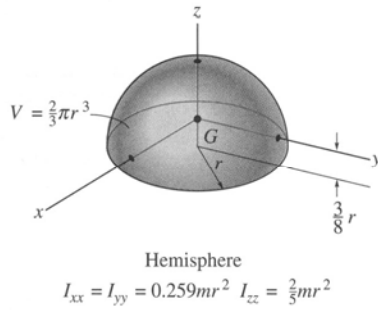
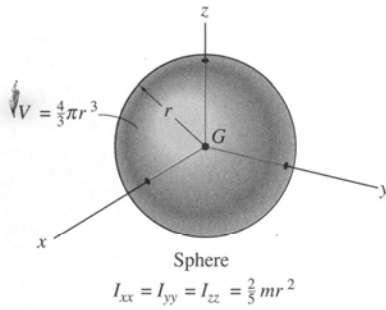
<p><b>Relative Motion Equations for a Rigid Body:</b></p>	$\bar{v}_A = \bar{v}_B + (\bar{v}_{A/B})_{rel} = \bar{v}_B + \bar{\omega}_{AB} \times (\bar{r}_{A/B})_{rel}$ $\bar{a}_A = \bar{a}_B + (\bar{a}_{A/B})_{rel} = \bar{a}_B + \bar{\alpha}_{AB} \times (\bar{r}_{A/B})_{rel} + \bar{\omega}_{AB} \times (\bar{\omega}_{AB} \times (\bar{r}_{A/B})_{rel})$ $\bar{a}_A = \ddot{\bar{R}} + \frac{\partial^2 \bar{\rho}_A}{\partial t^2} + 2\bar{\omega} \times \frac{\partial \bar{\rho}_A}{\partial t} + \dot{\bar{\omega}} \times \bar{\rho}_A + \bar{\omega} \times (\bar{\omega} \times \bar{\rho}_A)$ $T = \frac{1}{2} m(\bar{v}_G \cdot \bar{v}_G) + \frac{1}{2} \bar{\omega} \cdot \bar{H}_G$ $I_{xx} = (I'_{x'x'})_G + m(y_G^2 + z_G^2) \qquad I_{xy} = (I'_{x'y'})_G + mx_G y_G$ $I_{yy} = (I'_{y'y'})_G + m(x_G^2 + z_G^2) \qquad I_{yz} = (I'_{y'z'})_G + my_G z_G$ $I_{zz} = (I'_{z'z'})_G + m(x_G^2 + y_G^2) \qquad I_{zx} = (I'_{z'x'})_G + mz_G x_G$ $\bar{H}_A = \bar{H}_G + \bar{r}_{G/A} \times m\bar{v}_G$ $\bar{H}_A = (I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z)\hat{i} + (I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z)\hat{j} + (I_{zz}\omega_z - I_{xz}\omega_x - I_{yz}\omega_y)\hat{k}$
<p><b>Lagrange's Equations:</b></p>	$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} + \frac{\partial V}{\partial q_j} = Q_j \qquad \delta U = \sum Q_j \delta q_j$
<p><b>Lagrange Multipliers:</b></p>	$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = \sum \lambda_i a_{ij} + Q_j$ $\sum a_{jk} \dot{q}_k + b_j = 0$
<p><b>Hamilton's Canonical Equations:</b></p>	$H = \sum p_i \dot{q}_i - L$ $\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad p_n = \frac{\partial T}{\partial \dot{q}_n} = \frac{\partial L}{\partial \dot{q}_n}$ $\dot{p}_i = -\frac{\partial H}{\partial q_i} + Q_i$

Name \_\_\_\_\_

**Qualifying Exam: Dynamics & Vibrations**

**CLOSED BOOK**

**Center of Gravity and Mass Moment of Inertia of Homogeneous Solids**



Name \_\_\_\_\_

Spring 2010

## Qualifying Exam: Dynamics and Vibrations

### CLOSED BOOK

---

This portion of the qualifying exam is closed book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #\_\_\_\_, #\_\_\_\_, and #\_\_\_\_ graded.

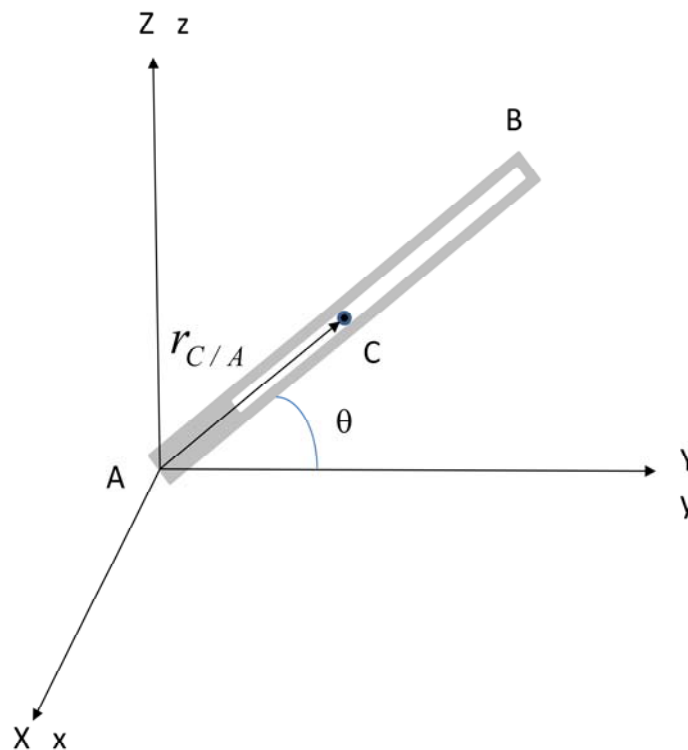
Be sure to put your name on all papers handed in, including this cover sheet.



## Qualifying Exam: Dynamics and Vibrations

**CLOSED BOOK**

1. Particle  $C$  moves along the smooth slot of  $AB$  at a constant rate of  $\dot{r}_{C/A}$  ( $m/s$ ) while the slotted rod rotates about  $Z$ -axis with a constant angular speed of  $\omega_1$  ( $rad/s$ ) and it is also rotating about  $x$ -axis with a constant angular speed of  $\omega_2$  ( $rad/s$ ).  $XYZ$  is a coordinate system fixed in space and the  $xyz$  coordinate system is welded to rod  $AB$ . At the instant shown,  $xyz$  and  $XYZ$  coincide and the distance of particle  $C$  from point  $A$  is  $r_{C/A}$  ( $m$ ). Determine the velocity and the acceleration of particle  $C$  at the instant shown.

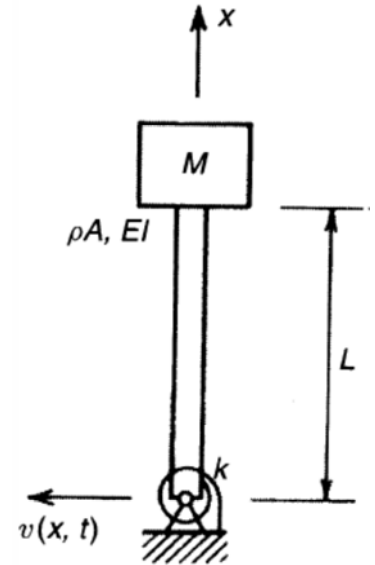


## Qualifying Exam: Dynamics and Vibrations

### CLOSED BOOK

2. An offshore oil-drilling platform is modeled as a uniform flexible beam of length  $L$  with a lumped mass  $M$  at the top and a rotational spring  $k$  at the bottom (see the figure on the left). Derive the Lagrange's equations of motion of a 2-DOF model of this structure by using the assumed-modes method. That is, one assumes that  $v(x,t) \approx \phi_1(x)q_1(t) + \phi_2(x)q_2(t)$ , where a polynomial function can be used as an admissible function for the (mode) shape function. Here, an admissible function implies the solution of the partial differential equation of motion for a continuous system which satisfies at least the given boundary conditions; and in this problem, one may try  $\phi_n(x) = (x/L)^n, n = 1, 2, \dots$ . Note that, using  $\dot{v} \square \partial v(x,t) / \partial t$  and  $v' \square \partial v(x,t) / \partial x$ , the kinetic and potential energies for this system can be written as

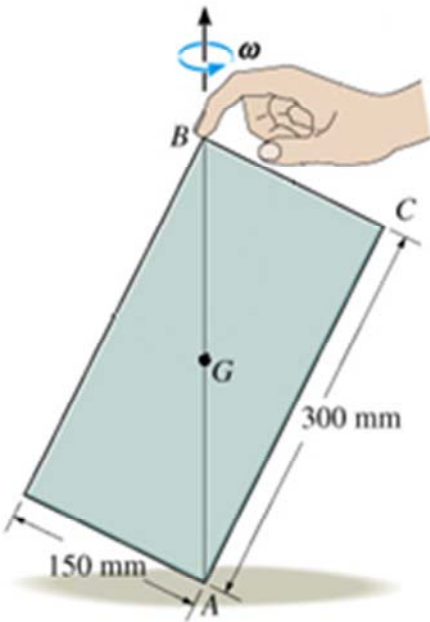
$$T = \frac{1}{2} \int_0^L \rho A \dot{v}(x,t)^2 dx + \frac{1}{2} M \dot{v}(L,t)^2, \quad V = \frac{1}{2} \int_0^L EI v''(x,t)^2 dx + \frac{1}{2} k \dot{v}(0,t)^2$$



## Qualifying Exam: Dynamics and Vibrations

## CLOSED BOOK

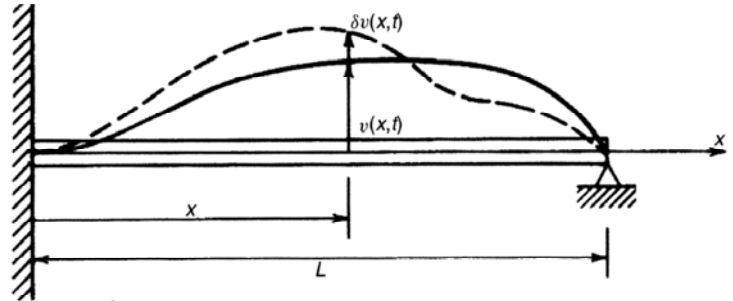
3. A thin uniform plate having a mass of  $0.4 \text{ kg}$  is spinning with a constant angular velocity  $\omega$  about its diagonal  $AB$ . If the person holding the corner of the plate at  $B$  releases his finger, the plate will fall downward on its side  $AC$ . Determine the necessary couple moment  $M$  which if applied to the plate would prevent this from happening.



## Qualifying Exam: Dynamics and Vibrations

## CLOSED BOOK

4. Consider an Bernoulli-Euler beam of length  $L$ , which is uniform and homogeneous. That is,  $m = \text{constant}$  and  $EI = \text{constant}$  along the beam length where  $m = \rho A$ ,  $E$  and  $I$  are the line density, Young's modulus and area moment of inertia about  $z$ -axis. The beam possesses fixed-hinged boundary conditions, and we want to study the free vibrations of the beam in the  $y$ -direction.



- a. The kinetic and potential energies of the beam can be expressed as

$$T = \frac{1}{2} \int_0^L m \dot{v}^2 dx, \quad V = \frac{1}{2} \int_0^L EI v''^2 dx$$

- where  $\dot{v} \equiv \partial v(x,t)/\partial t$  and  $v' \equiv \partial v(x,t)/\partial x$ . Knowing that the extended Hamilton's principle for a time interval  $[t_1, t_2]$  is written as  $\delta \int_{t_1}^{t_2} (T - V) dt = 0$ , derive the partial differential equation of motion for the transverse vibrations with corresponding four boundary conditions.
- b. Finding the solutions of the governing equation of motion above (i.e., by means of separation of variables), express the free vibrations of the beam in terms of the normal modes. Also, sketch the mode shape functions up to the 3<sup>rd</sup> mode (Note: The characteristic features must be depicted for full credit).
- c. Assuming that the beam possesses distinct natural frequencies, show that the corresponding normal modes are orthogonal with  $m(x) = m$  and  $EI(x) = EI$  being weight functions.

**Qualifying Exam: Dynamics and Vibrations****CLOSED BOOK**

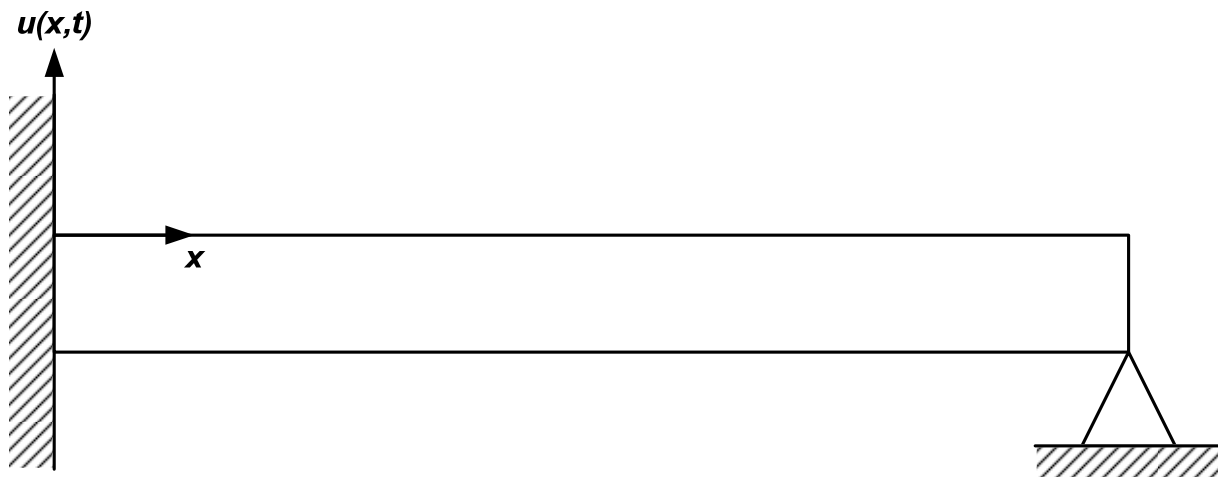
1. An Euler-Bernoulli beam of length  $l$ , of density per unit length  $\rho$ , and bending stiffness  $EI$  has transverse free vibration governed by the equation

$$EI \frac{\partial^4 u}{\partial x^4} + \rho \frac{\partial^2 u}{\partial t^2} = 0$$

If the beam is fixed at one end and pinned at the other, then

- Write the boundary conditions for both ends.
- Find the transcendental equation for the natural vibration frequencies and the corresponding mode shapes.
- Using the boundary conditions, sketch the first 3 mode shapes.

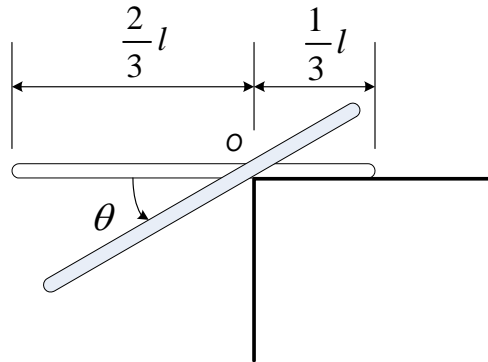
(Note: You do not have to solve the frequency equation.)



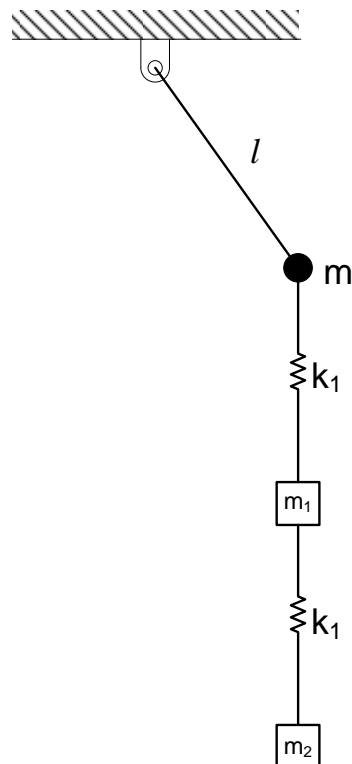
## Qualifying Exam: Dynamics and Vibrations

**CLOSED BOOK**

2. The uniform bar has a mass  $m$  and length  $l$ . If it is released from rest when  $\theta = 0^\circ$  (at an edge of a table as shown in the figure), determine the angle  $\theta$  at which it first begins to slip. The coefficient of static friction at the contact point  $O$  is  $\mu_s = 0.3$ .



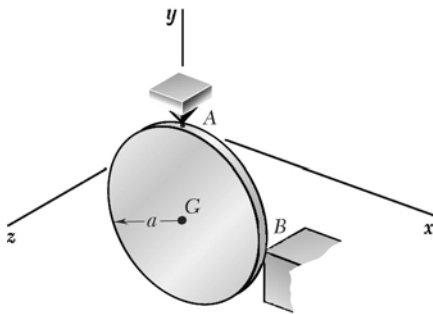
3. Determine the linearized equations of motion for the system shown.



## Qualifying Exam: Dynamics and Vibrations

## CLOSED BOOK

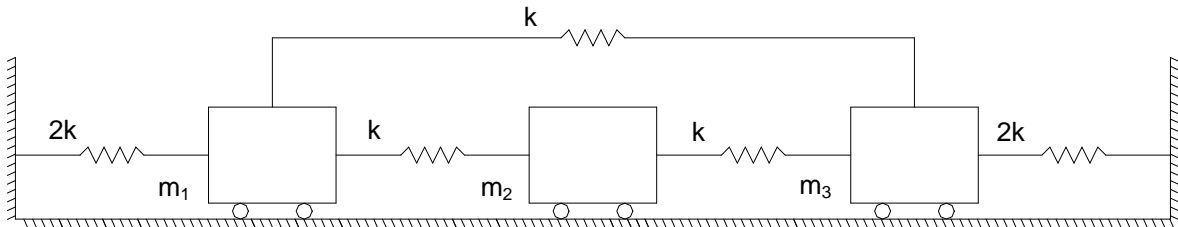
4. A circular plate of radius  $a$  and mass  $m$  supported by a ball-and-socket joint at  $A$  is rotating about the  $y$  axis with a constant angular velocity  $\omega = \omega_0 \mathbf{j}$  when an obstruction is suddenly introduced at point  $B$  in the  $xy$  plane. Assuming the impact at point  $B$  to be perfectly plastic ( $e = 0$ ), determine immediately after the impact (a) the angular velocity of the plate, (b) the velocity of its mass center  $G$ .



## Qualifying Exam: Dynamics &amp; Vibrations

**CLOSED BOOK**

1. For the system below, determine the natural frequencies and mode shapes.

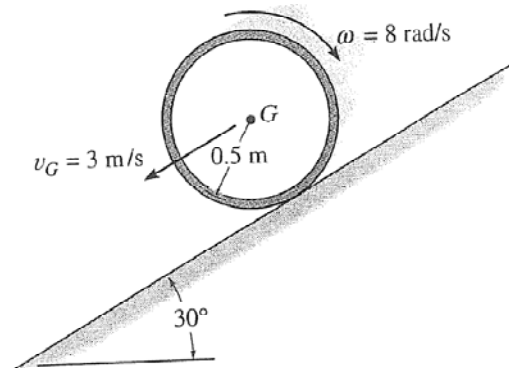


Given:

$$k = 4500 \text{ N/m}$$

$$m_1 = m_2 = m_3 = 25 \text{ kg}$$

2. The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin  $\omega = 8 \text{ rad/s}$  and its center has a velocity  $v_G = 3 \text{ m/s}$  as shown. If the coefficient of kinetic friction between the hoop and the plane is  $\mu_k = 0.6$ , determine how long the hoop rolls before it stops slipping.



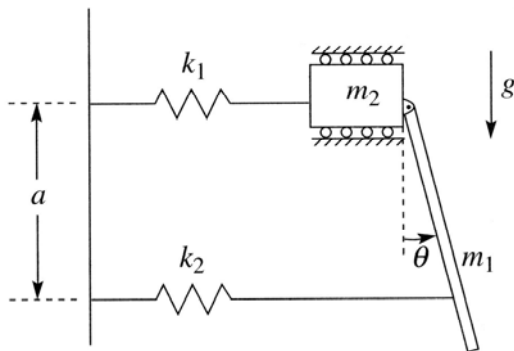


## Qualifying Exam: Dynamics &amp; Vibrations

## CLOSED BOOK

3. Find the equations of motion for the system below. The bar is of length  $L$  and  $a = \frac{3L}{4}$ .

$$I = \frac{1}{12} mL^2$$



4. The disk has a mass  $m$  and radius  $r$ . If it strikes the rough step having a height of  $(1/8)r$  as shown, determine the largest angular velocity  $\omega_1$  the disk can have (at the beginning of the striking) and not rebound off the step when it strikes it.  
[Hint: non-rebounding requires zero normal force at the impact point right after the strike]

