Controls Problems for Qualifying Exam - Spring 2014

Problem 1

Consider the system block diagram given in Figure 1. Find the overall transfer function $T(s) = C(s)/R(s)$. Note that this transfer function will have a quadratic polynomial of the form $D(s) = s^2 + 2\zeta\omega_{\mathrm{n}}s + \omega_{\mathrm{n}}^2$ in the denominator. Determine the value of k such that the damping ratio ζ is 0.5. Then obtain the rise time t_r to an unit-step reference, which is given by $t_r = \dfrac{\pi - \beta}{\omega_{\mathrm{d}}}$, where $\cos\beta = \zeta$ (β in radians) and $\omega_{\mathrm{d}} = \omega_{\mathrm{n}}\sqrt{1-\zeta^2}$. Also obtain the numerical values of the

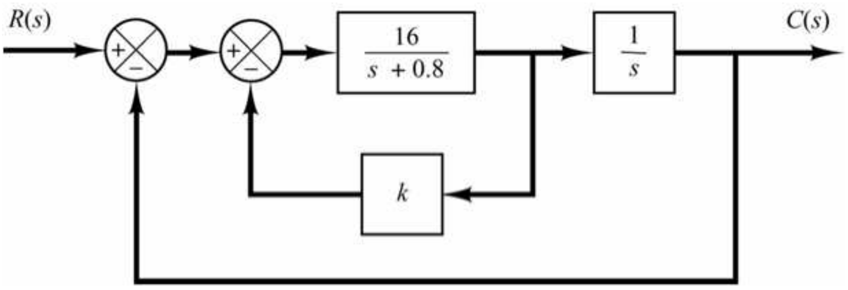


Figure 1: *Block diagram of system.*

peak time t_p and the maximum overshoot M_p, which are given by the formulas (9)

$$t_p = \frac{\pi}{\omega_{\mathrm{d}}} \quad \text{and} \quad M_p = e^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}.$$

Problem 2

Consider the following linear time-invariant system in state space form:

$$\begin{aligned}\dot{\mathbf{x}} &= A\mathbf{x} + Bu,\\ y &= C\mathbf{x}, \quad \text{with } A = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = [0 \;\; 1].\end{aligned}$$

where $\mathbf{x} = [x_1 \;\; x_2]^{\mathrm{T}}$ is the state vector, u is the system input, and y is the system output. It is desired to place the closed-loop poles at $s = -2 \pm \jmath 2$. Determine the required linear state variable feedback control law, assuming full state feedback.

Problem 3

The following polynomial is the characteristic equation of a closed-loop control system:

$$a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0.$$

Assuming the coefficient a_4 is a nonzero positive number and $a_0 > 0$. Determine the conditions of the coefficients of the equation such that the control system is stable. The resulting conditions must be reduced to the simplest polynomials.

Problem 4

Assume $\mathbf{A}$ is a state matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}.$$

Compute the exponent matrix $e^{\mathbf{A}t}$ where t is the independent variable representing time.

[Hint: You may use the Sylvesters interpolation formula $e^{\mathbf{A}t} = \alpha_0(t)\mathbf{I} + \alpha_1(t)\mathbf{A} + \alpha_2(t)\mathbf{A}^2$ to solve the problem. You may also use another method such as $e^{\mathbf{A}t} = \mathbf{E}e^{\mathbf{D}t}\mathbf{E}^{-1}$.]

Qualifying Exam Control Problems (Fall 2013)

(solve only 3 from the following 4 problems!!)

1. Consider a lag-lead compensator $G(s)$ defined by

$$G(s) = K\frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{\beta}{T_1}\right)\left(s+\frac{1}{\beta T_2}\right)}$$

Assume T_1 and T_2 are nonzero. Find an $\omega \neq 0$ such that at ω the phase angle of the compensator is zero.

[Hint: consider that $x = y$ is equivalent to $\tan x = \tan y$]

2. Obtain the response of the following linear system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

assuming that the control input $u(t)$ is a unit-step function and the initial condition of the system is $\mathbf{x}(0) = \mathbf{0}$.

3.

Consider a unity feedback control system whose open-loop transfer function is

$$L(s) = \frac{K}{s(Js+B)}.$$

Discuss the effects that varying the values of K and B have on the steady-state error in unit-ramp response (i.e., $R(s) = \frac{1}{s^2}$). Sketch typical unit-ramp response curves for a small value, a medium value, and a large value of K, assuming that B is constant.

4.

Show that the transfer function of a linear time-invariant single input single output (SISO) system is not changed by a linear (non-singular) transformation of the state. Consider the original system to be of the form

$$\begin{aligned} \dot{\mathrm{x}} &= A\mathrm{x} + Bu, \\ y &= C\mathrm{x} + Du, \end{aligned}$$

where $\mathrm{x} = [x_1 \ldots x_n]^{\mathrm{T}}$ is the state vector, u is the system input, and y is the system output.

Name ______________________________ **Spring 2010**

Qualifying Exam: Controls

CLOSED BOOK

This portion of the qualifying exam is closed book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

1. A control system is to be designed to control the short period mode of the next generation stealth fighter (the F44 Chupacabra). The initial design is to be done using an approximation defined by a second order model. For the design flight condition, the numerical values to be used in the model are shown in the equation below, where α and q are the two states and u is the control.

$$\begin{Bmatrix} \dot{\alpha} \\ \dot{q} \end{Bmatrix} = \begin{bmatrix} -0.5 & 1 \\ -2 & 1.5 \end{bmatrix} \begin{Bmatrix} \alpha \\ q \end{Bmatrix} + \begin{Bmatrix} 0 \\ -2.5 \end{Bmatrix} u$$

   a. For the numerical values given, determine whether the math model for the uncontrolled airplane (u = 0) yields a stable or an unstable motion.

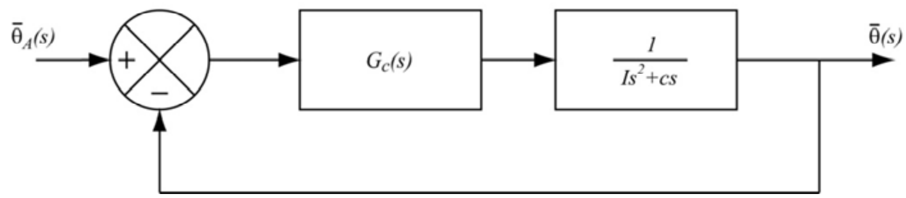
   b. Use full state feedback to determine control constants that will move the eigenvalues such that the overshoot (in response to a step input) is 30% and the oscillation frequency is $3\ rad/sec$.

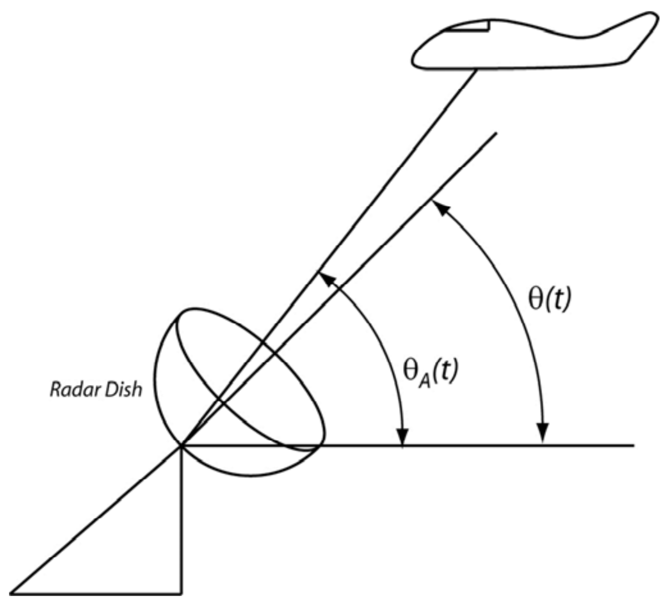
Name ______________________________ **Spring 2010**

Qualifying Exam: Controls

CLOSED BOOK

2. A radar tracking dish is to track an airplane, such that if the airplane moves across the sky at the fixed angular rate $\theta_A(t) = 3°/s$, the steady-state tracking error is $0.1°$ or less. In addition, the largest system time constant should be less than or equal to $0.5s$. A PD controller is proposed to control the radar dish motion, $G_c(s) = k_p + k_d s$. If $I = 20$, $c = 5$, $k_p = 10$, and $k_d = 1$, all in appropriate units, determine whether the design specifications are realized.





Name ______________________________ **Spring 2010**

Qualifying Exam: Controls

CLOSED BOOK

---

3. The differential equation of a linear SISO system is

$$\frac{d^4y(t)}{dt^4} + 4\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + y(t) = r(t) + 2\frac{dr(t)}{dt}$$

where y(t) is the output and r(t) is the input.

a. Write the transfer function $Y(s)/R(s)$.

b. Draw the block diagram for the **controllable canonical form** and write the corresponding state space equations. Be sure to label all states in the block diagram.

c. Draw the block diagram for the **observable canonical form** and write the corresponding state space equations. Be sure to label all states in the block diagram.

d. Find the steady state value of $y(t)$ if $r(t)$ is a unit step input using the final value theorem.

e. If the plant model above is subjected to negative unity feedback, derive an expression for the error $E(s) = R(s) - Y(s)$ and find the steady-state error to a unit step input using the final value theorem.

Name ______________________________ **Spring 2010**

Qualifying Exam: Controls

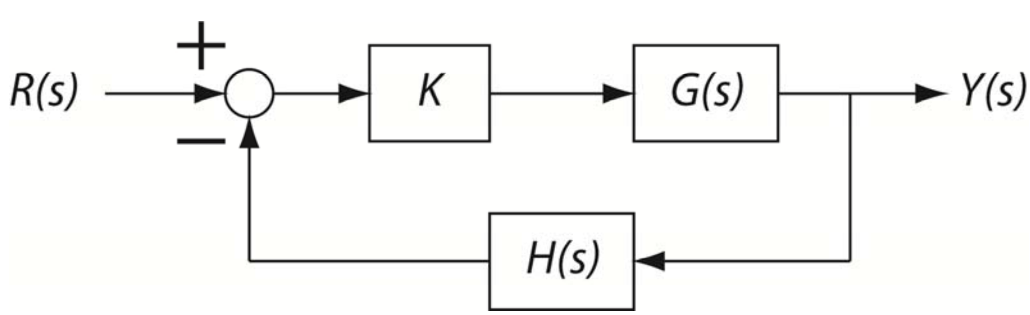
CLOSED BOOK

4. For the closed loop system shown below:
    a. Define the root locus of a closed loop system in terms of the characteristic equation. How does the root locus vary with K in terms of the poles and zeros of $G(s)H(s)$?
    b. What portions of the real axis are on the root locus?
    c. What is the rule for the asymptotic angles as K approaches infinity?
    d. What is the rule for the centroid of the asymptotes?
    e. What is the rule for breakaway points in terms of K?
    f. What are the rules for the angles of departure and approach?
    g. How are the values of K at which the locus crosses the imaginary axis determined?
    h. Sketch the root locus for:

$$i. \qquad G(s)H(s) = \frac{s+0.5}{s(s+1)(s+2)}$$

$$ii. \qquad G(s)H(s) = \frac{s+2}{(s+1)(s+0.5)}$$

$$iii. \qquad G(s)H(s) = \frac{s}{(s+0.5)(s+1)(s+2)}$$



Name ______________________________ **Fall 2008**

Qualifying Exam: Controls

CLOSED BOOK

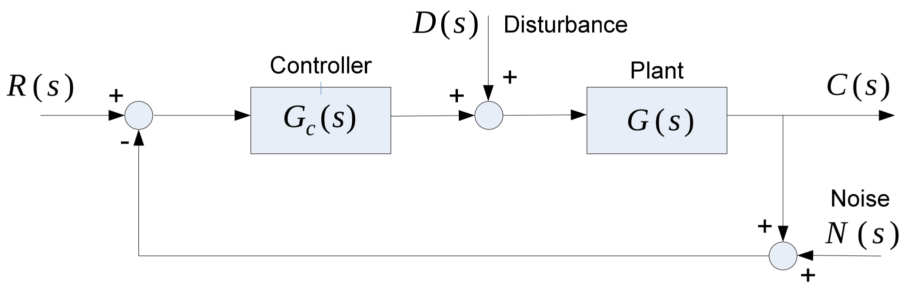
This portion of the qualifying exam is **closed** book. You may have a calculator.

Work 3 of the 4 problems. Be very clear which 3 you want graded (see below). It is not acceptable to work all 4 problems and hope that the graders pick out the best worked three.

I want problems #____, #____, and #____ graded.

Be sure to put your name on all papers handed in, including this cover sheet.

1. The following block diagram represents a closed-loop linear control system



Circle the correct answer to each of the following ten questions. Note: multiple circles to one question will be considered as a wrong answer.

1.1. In order to have a better disturbance-rejection capability, the gain of the controller $G_c(s)$ should be

a) large; b) small; c) does not matter.

1.2. In order to have a better noise-attenuation capability, the gain of the controller $G_c(s)$ should be

a) large; b) small; c) does not matter.

1.3. If a point *s* (which is a complex number in the *s* plane) satisfies the conditions

$$< G_c(s)G(s) = 180^\circ \text{ and } |G_c(s)G(s)| = 1$$

then it is: a) an open-loop zero; b) a closed-loop zero; c) an open-loop pole; d) a closed-loop pole

1.4. Is any point of the root loci of $G_c(s)G(s)$ a closed-loop pole of the system?

a) yes; b) no.

Name ______________________________ **Fall 2008**

Qualifying Exam: Controls

CLOSED BOOK

1.5. When the open-loop gain K varies from 0 to ∞, the root locus goes
   a) from a zero to a pole;
   b) from a pole to a zero or infinity;
   c) depending.

1.6. The component of a PID controller which can help reduce the steady-state error is
   a) the proportional component;
   b) the integral component;
   c) the differential component;
   d) all of them.

1.7. Which of the following techniques is best for analyzing the transient response of a control system:
   a) Root-locus; b) Bode diagram; c) Nyquist diagram; d) any one of these

1.8. For a minimum-phase control system to be stable, its Nyquist diagram must
   a) clockwise encircle (-1, 0) of the s plane once;
   b) counterclockwise encircle (-1, 0) of the s plane once;
   c) not encircle (-1, 0) of the s plane;
   d) depending;

1.9. When designing a control system, which of the following phase margin is most appropriate
   a) 0º; b) 15º; c) 45º; d) 90º

1.10. Assume the close-loop transfer function of the system is $\frac{K(s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m)}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$ and the system is stable, then the coefficients of the denominator, $a_1, a_2, \cdots, a_n$,
   a) must be all negative; b) must be all positive;
   c) can be positive or negative depending on the transfer function

Name ______________________________ **Fall 2008**

Qualifying Exam: Controls

CLOSED BOOK

2. Given the closed-loop transfer function

$$T(s) = \frac{Y(s)}{U(s)} = \frac{2s^2 + 8s + 6}{s^3 + 8s^2 + 16s + 6}$$

   a. Draw a block diagram for the system in which each block is either a constant gain or an integration.

   b. Write the state space model corresponding to the block diagram in a. in the form

$$\dot{x} = Ax + Bu(t)$$
$$y(t) = Cx + Du(t)$$

   (specifying the matrices A,B,C, and D) where *x(t), u(t)*, and *y(t)* are the state vector, input vector, and output vector, respectively.

3. A feedback control system has a characteristic equation

$$s^3 + (4 + K)s^2 + 6s + (16 + 8K) = 0$$

   a) Use the Routh-Hurwitz stability criterion to find the maximum value of *K* for stability.

   b) For *K* set equal to the maximum value, what are the resulting three roots?

Name ______________________________ **Fall 2008**

Qualifying Exam: Controls

CLOSED BOOK

4. The following control system involves a velocity feedback. Determine the amplifier gain K and the velocity feedback gain K_h such that the closed-loop system
   a. is critically damped and
   b. has an undamped natural frequency of 2 rad/s.

   What is the condition on K and K_h such that the system is stable?