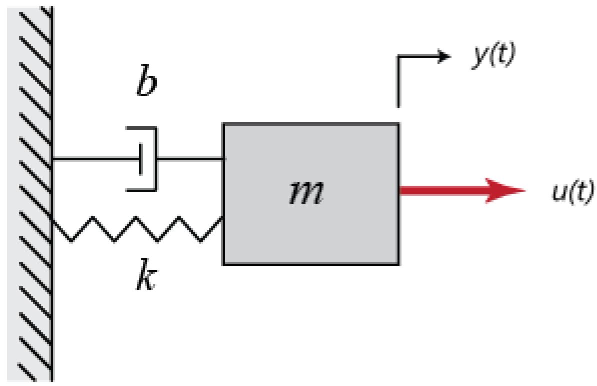
Qualifying Exam, Fall 2021
Control

* This is a closed-book test (with a cheat sheet provided), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #____________, #___________, and #____________ to be graded.

Problem 1. Consider the mass-spring-damper system below with the mass, $m = 1$ kg, damper constant, , $b = 3$ Ns/m, and spring constant $k = 2$ N/m. Define the system input force as $u(t)$ and the displacement of the mass as $y(t)$.



(1) Defining the system state vector as $\mathbf{x} = [y(t) \;\; \dot{y}(t)]^T$, find the state-space equations of the system. (4 pts)

(2) Is the system controllable? Why or Why not? (3 pts)

(3) Is the system observable? Why or Why not? (3 pts)

Problem 2. Consider the linear system $\dot{x} = Ax + Bu,\ y = Cx$, where

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0]$$

(1) Is the matrix A asymptotically stable, marginally stable, or unstable? Why or Why not? (5 pts)

(2) Is the system bounded-input-bounded-output (BIBO) stable? Why or why not? (5 pts)

Problem 3. Find $Y(s)$ where $u(t) = e^{-t}\cos t$. Refer to the Laplace Transform Pairs table.

(a) $G(s) = \frac{3}{s(s+4)(s+5)}$ (5 pts)

(b) $G(s) = \frac{2(s+3)}{s(s^2+2s+2)}$ (5 pts)

Problem 4. Derive the transfer functions for the following systems. For each system, find the zero-initial condition responses to $\sin 3t$. Refer to the Laplace Transform Pairs table.

(1) $\dot{x} = \begin{bmatrix} -0.2 & 0 \\ -1 & 0.8 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} -1 & 1 \end{bmatrix} x.$ (5 pts)

(2) $\dot{x} = \begin{bmatrix} -5 & 1 \\ -6 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x.$ (5 pts)

Table A–1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

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Table A–1 *(continued)*

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t \quad (0 < \zeta < 1)$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t - \phi\right)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, \; 0 < \phi < \pi/2)$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \phi\right)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, \; 0 < \phi < \pi/2)$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
25	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
26	$\omega t - \sin \omega t$	$\frac{\omega^3}{s^2(s^2+\omega^2)}$
27	$\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
28	$\frac{1}{2\omega} t \sin \omega t$	$\frac{s}{(s^2+\omega^2)^2}$
29	$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2}(\cos \omega_1 t - \cos \omega_2 t) \quad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2+\omega_1^2)(s^2+\omega_2^2)}$
31	$\frac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$	$\frac{s^2}{(s^2+\omega^2)^2}$