Qualifying Exam, Fall 2021
Fluid Mechanics

* This is a closed-book test (with a cheat sheet provided), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #____________, #___________, and #____________ to be graded.

Problem 1. The laminar boundary layer profile can be approximated by

$$u = \begin{cases} U\left[1 - \left(\frac{y}{\delta} - 1\right)^2\right] & y < \delta \\ U & else \end{cases}$$

Here, δ is the boundary layer thickness. The freestream velocity shall be constant: $U = const.$ and $\frac{\partial U}{\partial x} = 0$.

a) Provide an expression for the displacement thickness, δ^*, as a function of δ.
b) Provide an expression for the momentum thickness, ϑ, as a function of δ.
c) Provide an expression for the skin-friction coefficient

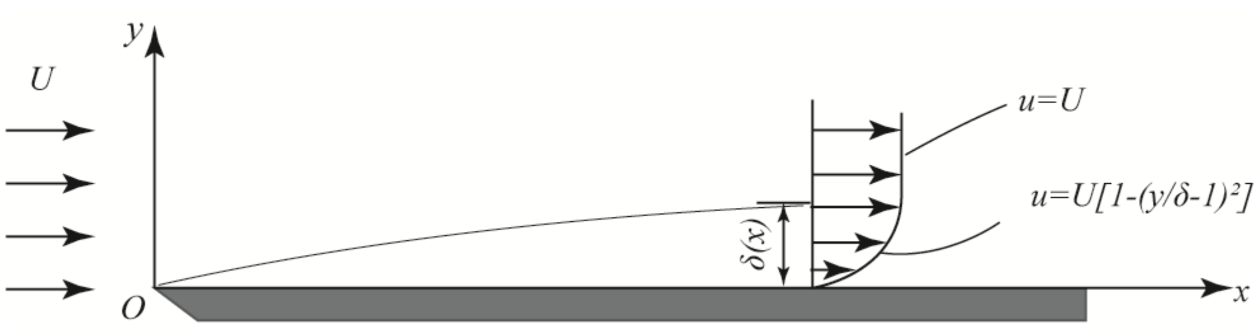
$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$$

as a function of the boundary layer thickness Reynolds number, $Re_\delta = \frac{\rho U \delta}{\mu}$. The wall friction is $\tau_w = \mu \frac{\partial u}{\partial y}$.

The x-momentum equation can be integrated in y to obtain

$$\frac{1}{\rho}\tau_w = \frac{d}{dx}(U^2\vartheta) + U\delta^* \frac{dU}{dx}$$

d) From this equation, derive an expression for the skin-friction coefficient, c_f, as a function of δ.
e) Set expressions for c_f from (c) and (d) equal to each other and find expression for Re_δ as a function of Re_x.
f) Provide expressions for $\frac{\delta}{x}$ and c_f as a function of Re_x.



Problem 2. Consider a freestream in x that is superimposed with a dipole that is rotated by 90deg,

$$\Psi = U\left(y + \frac{a^2 x}{x^2 + y^2}\right)$$

The freestream velocity shall be $U = 1$ and the dipole strength shall be $a^2 = 1$. Because of the dipole rotation, the resulting potential flow is NOT a model for the flow around a circular cylinder.

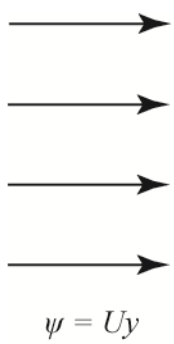
a) Provide expressions for u and v as function of x and y.

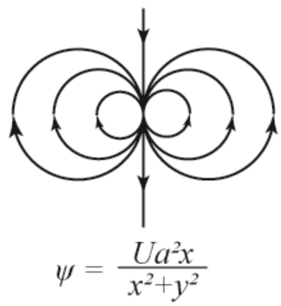
b) Provide expressions for $v_r = \frac{1}{r}\frac{\partial \Psi}{\partial \vartheta}$ and $v_\vartheta = -\frac{\partial \Psi}{\partial r}$ as function of r and ϑ.

c) The flow has two stagnation points (saddle points). Provide r and ϑ locations for stagnation points.

d) Provide analytical expression, $r = r(\vartheta)$, for streamlines passing through one of the stagnation points.

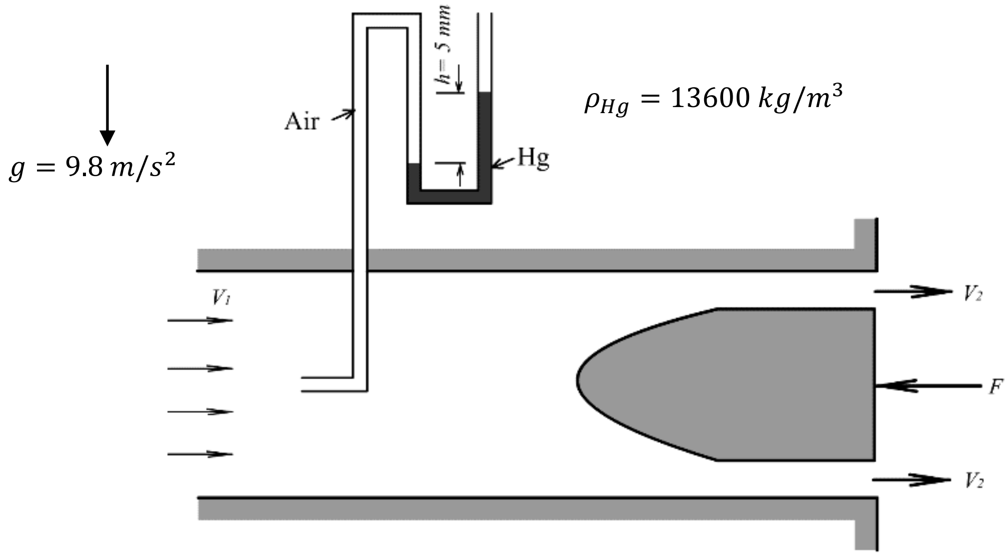
e) Provide a rough sketch of the streamlines for the flow field (pencil and paper).





Problem 3. Air flows through a circular pipe with inside diameter of 0.5 m ($D_1 = 0.5\ m$). A smoothly contoured plug of 0.4 m diameter ($D_2 = 0.4\ m$) is held in the end of the pipe where air discharges to atmosphere (see figure below). Air density is $\rho = 1.3\ kg/m^3$ and considered incompressible. Neglect the friction effects and assume uniform velocity profile at each cross-section. Determine:

a) The mass flowrate though the pipe.
b) Magnitude of force, F, required to hold the plug.

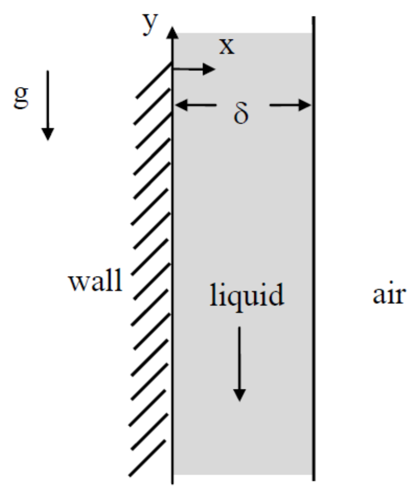


COM: $\frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$

COLM: $\frac{\partial}{\partial t}\int_{CV} \rho \vec{V} d\forall + \int_{CS} \rho \vec{V}\left(\vec{V} \cdot d\vec{A}\right) = -\int_{CS} p d\vec{A} + \vec{F}$

Problem 4. A viscous, incompressible, Newtonian liquid flows in steady, laminar, planar flow down a vertical wall. The thickness, δ, of the liquid film remains constant. Since the liquid free surface is exposed to atmospheric pressure, there is no pressure gradient in the liquid film. Furthermore, the air provides a negligible resistance to the motion of the fluid. Fluid density is ρ and dynamic viscosity is μ. Assume flow is fully developed ($\frac{\partial \vec{V}}{\partial y} = 0$).

a) Determine the velocity distribution for this gravity driven flow. Clearly state all assumptions and boundary conditions.
b) Determine the shear stress acting on the wall by the fluid.
c) Determine the maximum velocity magnitude of the flow.



Continuity equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Navier-Stokes Equations:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$