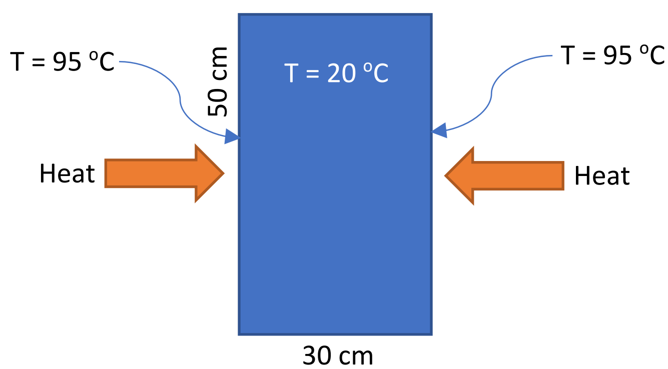
Fall 2021 Qualifying Exam – Thermal Sciences Q1 – Conduction Heat Transfer

A person intends to kill coronavirus on the surfaces of a thin solid plate in a machine shop by adding thin non-uniform resistive heating strips on its sides. The plate has a width of 30 cm (*X* direction) and a height of 50 cm (*Y* direction) and it is initially at a temperature 20 °C. If the person heats the two sides of the plate as shown below which suddenly increases their temperature to 95 °C everywhere along the height, develop an analytical solution to predict the temperature distribution in the plate as a function of time *t*. Assume: there are no temperature gradients in the height and the thickness directions, the thermal diffusivity of the plate material is α, and the Biot number is 1 (8.5 points).

If it is hypothetically true that the coronavirus can be instantly killed at temperatures above 85 °C, use the developed solution to "suggest" a "theoretical framework" (describe in words but do not solve for anything) for predicting the distance from the hot sides of the plate until where the coronavirus might have died after 5 minutes of heating the plate (1.5 points).

Heat Conduction Equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho\cdot c_p\frac{\partial T}{\partial t}$$



$$\frac{\partial^2 X}{\partial x^2}+\lambda^2 X=0\ ;\ \text{solution: } X(x)=A\cos(\lambda x)+B\sin(\lambda x)$$

$$\frac{\partial^2 X}{\partial x^2}-\lambda^2 X=0\ ;\ \text{Solution: } X(x)=Ae^{-\lambda x}+Be^{\lambda x}$$

$$\frac{\partial X}{\partial t}+c\lambda^2 X=0\ ;\ \text{Solution: } X(t)=De^{-c\lambda^2 t}$$

$$\sin^2(x)=\frac{1-\cos(2x)}{2}$$

Fall 2021 Qualifying Exam – Thermal Sciences Q2 – Convection Heat Transfer

Hot water (at 80 ^{0}C) is flowing through a steel pipe (thermal conductivity of steel is 43 W/m·K) with a velocity of 0.05 m/s. The pipe is 10 m long and has an inside diameter of 5 cm and an outside diameter of 15 cm. Air (at 20 ^{0}C) on a windy day is moving across the pipe (perpendicular to the pipe axis) on its outside surface with a velocity of 25 m/s. Calculate the heat loss from the hot water to air (10 points).

For water, use density=990 kg/m^3, specific heat=4175 J/kg·K, absolute viscosity=0.66x10^{-3} Pa·s, Prandtl number=4.3 and thermal conductivity=0.63 W/m·K.

For air, use density=1.092 kg/m^3, specific heat=1014 J/kg·K, Prandtl number=0.71, absolute viscosity=19.123x10^{-6} Pa·s, and thermal conductivity=0.0265 W/m·K.

External Flow

Cylinder in Cross-Flow

$Re_{cr} \cong 2 \times 10^5$

$$Nu(\theta) = 1.14 \times Re_D^{0.5} \times Pr^{0.4} \times \left[1 - \left(\frac{\theta}{90}\right)^3\right] \quad \theta < 90^\circ$$

Laminar flow

$$\overline{Nu}_D = 0.012 \times Re_D^{0.85} \times Pr^{0.36}$$

Turbulent flow

Internal Flow

$$\overline{Nu}_{D_h} = \begin{cases} \left[1.615 \times Gz^{-1/3}\right] - 0.7 & \text{if } Gz \le 0.005 \\ \left[1.615 \times Gz^{-1/3}\right] - 0.2 & \text{if } 0.005 < Gz < 0.03 \\ 3.657 + \dfrac{0.0499}{Gz} & \text{if } Gz \ge 0.03 \end{cases}$$

Gz = Graetz Number $= \dfrac{L}{D_h \cdot Re_{D_h} \cdot Pr}$

Internal laminar developing flow: constant wall temperature boundary condition

$$\overline{Nu}_{D_h} = \begin{cases} 1.953 \times Gz^{1/3} & \text{if } Gz \le 0.03 \\ 4.364 + \dfrac{0.0722}{Gz} & \text{if } Gz > 0.03 \end{cases}$$

Internal laminar developing flow: constant wall heat flux boundary condition

$Re_{cr} \cong 2100$

$$D_h = \frac{4 \times A_{flow}}{P_{wetted}}$$

Internal turbulent flow

Gnielinski – Petukhov Correlation

$$\overline{Nu}_{D_h} = \frac{\left(\frac{f}{8}\right) \times (Re_{D_h} - 1000) \times Pr}{1 + \left[12.7 \times \left(\frac{f}{8}\right)^{1/2} \times \left(Pr^{2/3} - 1\right)\right]}$$

$$f = \left\{\left[0.79 \times \ln(Re_{D_h})\right] - 1.64\right\}^{-2}$$

f → friction factor

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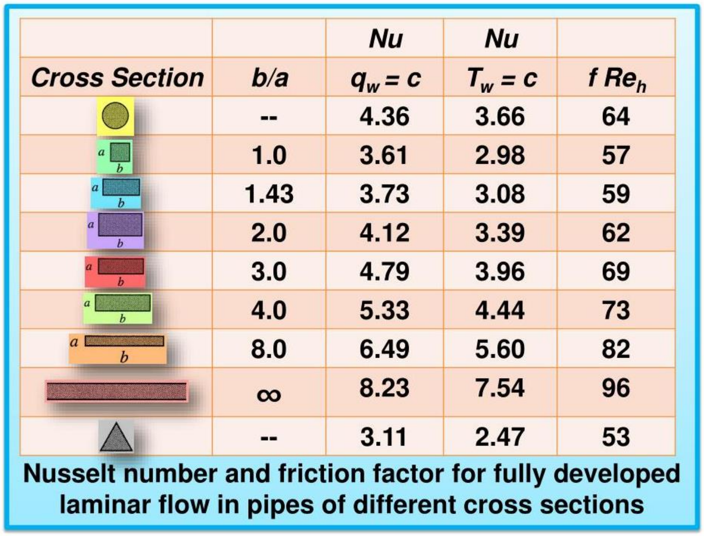


Cross Section	b/a	Nu $q_w = c$	Nu $T_w = c$	$f\ Re_h$
	--	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	∞	8.23	7.54	96
	--	3.11	2.47	53

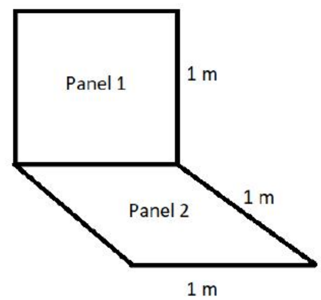
Nusselt number and friction factor for fully developed laminar flow in pipes of different cross sections

Fall 2021 Qualifying Exam – Thermal Sciences Q3 – Radiation Heat Transfer

All spacecraft use thermal radiators for rejecting the excess waste heat created by the on-board electronics and other systems. The radiators usually have various forms such as integrated structural panels or flat panels attached to the sides of the spacecraft.

Assume a spacecraft has a thermal radiator configuration of two flat panels arranged perpendicular to each other and joined by a common side as shown below. The size of each panel is 1 m by 1 m. Panel 1 (surface 1) is maintained at -50 ^{0}C (with an emissivity of 0.7) while Panel 2 (surface 2) is at 10 ^{0}C (with an emissivity of 0.8). Both the panels are exchanging heat with each other and with the surrounding outer space (surface 3), which is at 0 K and can be assumed as a black body.

Find: (i) the radiation leaving the surface of Panel 1 per unit area per unit time (5 points), and (ii) the radiation leaving the surface of Panel 2 per unit area per unit time (5 points).



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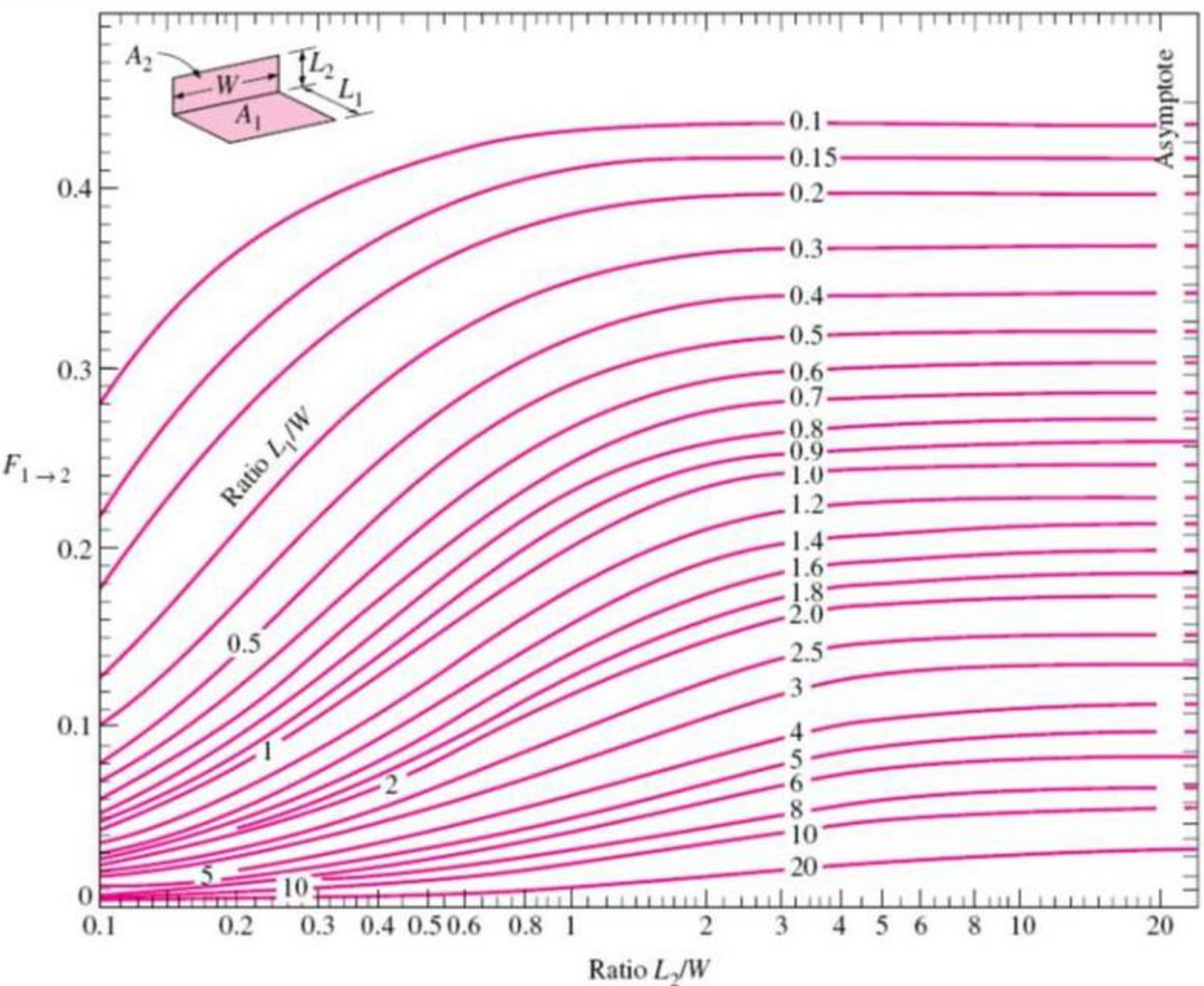

View factor between two perpendicular rectangles with a common edge.

Fall 2021 Qualifying Exam – Thermal Sciences Q4 – Thermodynamics

A rigid tank contains 2 kg of N_2 at 25 ^oC and 550 kPa. Another rigid tank contains 4 kg of O_2 at 25 ^oC and 150 kPa. A valve is connecting the two tanks. If this valve is suddenly opened and the two gases are allowed to mix, determine the volume of each of the two tanks and the final pressure of the mixture (7 points). Assume the final temperature of the mixture is 25 ^oC. The molar masses of N_2 and O_2 are 28.013 kg/kmol and 31.999 kg/kmol, respectively. The specific gas constants for N_2 and O_2 are 0.2968 kPa·m^3/kg·K and 0.2598 kPa·m^3/kg·K.

If the final temperature of the mixture is 50 ^oC instead, determine the percentage differences in the volume of each tank and the final pressure compared to the previous case (3 points).