Qualifying Exam 2021

Select three problems

Dynamics

- **Problem 1: (10 points)**

A constantly rotating disk is mounted on a moving train, as shown in Figure 1. The train itself is moving with a time-varying velocity $\mathbf{v}(\boldsymbol{t})=kV(t)\boldsymbol{d}_1$ where k is a constant.

1- Assume that the particle P is fixed on the disk at a radius r from its center,
   (a) what is its inertial velocity? Express your answer in **{d}** components as functions of r, ω, $\boldsymbol{k}, \boldsymbol{V}$, and α, where α is the angle between the DP line and $\mathbf{d}_1$. (2 points)
   (b) what is its inertial acceleration? Express your answer in **{d}** components as functions of r, ω,$\boldsymbol{k}, \dot{\boldsymbol{V}}$, and α, where α is the angle between the DP line and $\mathbf{d}_1$. (2 points)

2- Assume that the particle P is moving on the disk at a radius $r(t)$ from its center,
   (a) what is its inertial velocity? Express your answer in **{d}** components. (3 points)
   (b) what is its inertial acceleration? Express your answer in **{d}** components. (3 points)

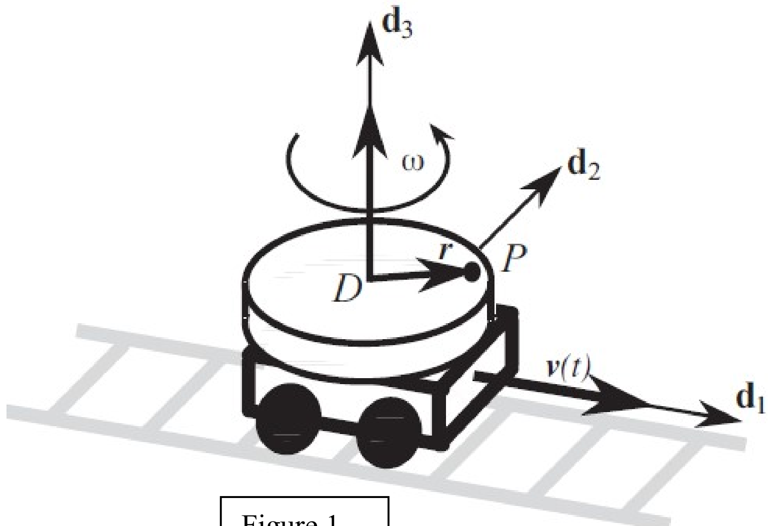

Figure 1

- **Problem 2: (10 points)**

A vertical disk of radius r is attached to a horizontal shaft of length R, as shown in Figure 2. The shaft is rotating at a time varying rate $\dot{\emptyset}$ about the vertical, while the disk is rotating about the horizontal shaft with at a time-varying rate $\dot{\theta}$. A fixed point P is on the rim of the disk, while a missile is flying overhead at a constant speed v_m in the negative $\mathbf{n_2}$ direction at a fixed height h with the trajectory $r_m = h\mathbf{n_3} - v_m t\mathbf{n_2}$

a) Demonstrate that the inertial velocity of point P can be expressed as (show all details): (4 points)

$$\dot{r} = \left(-r\dot{\emptyset}\sin\theta\right)\boldsymbol{e_r} + \left(R\dot{\emptyset} + r\dot{\theta}\cos\theta\right)\boldsymbol{e_\emptyset} + \left(-r\dot{\theta}\sin\theta\right)\boldsymbol{e_3}$$

b) Find the acceleration of point P. Express your answers completely in terms of E-frame, $\{\boldsymbol{e_r}, \boldsymbol{e_\emptyset}, \boldsymbol{e_3}\}$ components. (2 points)

c) What is the velocity of point P as seen by the missile? (2 points)

d) What is the acceleration of point P as seen by the missile? (2 points)

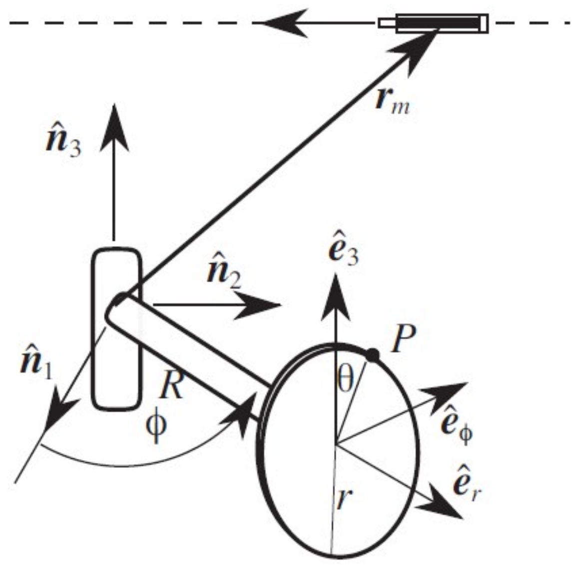

Figure 2

Vibrations

- **Problem 3: (10 points)**

For the dynamical system shown in Figure 3,

1- Demonstrate that the equations of motion of this system can be written as (show all details): (3 points)

$$(m+M)\ddot{x}+c\dot{x}+2kx+ml\ddot{\theta}\cos(\theta)-ml\dot{\theta}^2\sin\theta=F(t)$$

$$\ddot{\theta}+\frac{\cos(\theta)}{l}\ddot{x}+\frac{g}{l}\sin(\theta)=\frac{T(t)}{ml^2}$$

2- Linearize the equations of motion of this dynamical system and express them as a function of the mass ratio, $\mu = m/M$; and the normalized displacement of the mass M, $y = x/l$ as follows: (2 points)

$$\begin{bmatrix} 1+\mu & \mu \\ 1 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{y} \\ \ddot{\theta} \end{Bmatrix}+\begin{bmatrix} 2\xi_1\omega_1 & 0 \\ 0 & 2\xi_2\omega_2 \end{bmatrix}\begin{Bmatrix} \dot{y} \\ \dot{\theta} \end{Bmatrix}+\begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix}\begin{Bmatrix} y \\ \theta \end{Bmatrix}=\begin{Bmatrix} \dfrac{F(t)}{Ml} \\ \dfrac{T(t)}{ml^2} \end{Bmatrix}$$

3- What are the expressions of ξ_1, ξ_2, ω_1, and ω_2? (1 points)

4- Consider the damping coefficient zero (c=0),

(a) A harmonic excitation is applied to the mass M such that $F(t)=10F_0 \cos(\Omega t+\lambda)$ and $T(t)= 6F_0\, l \cos(\Omega t+\lambda)$. Find the expression of the length of the pendulum (l) as a function of the excitation frequency and gravitational acceleration that renders the mass M to be stationary under this harmonic excitation. (1.5 points)

(b) Determine the corresponding vibratory motion of the pendulum ($\theta(t)$). (0.5 points)

5- Consider the damping coefficient zero (c=0),

(a) A harmonic excitation is applied to the system such that $F(t)=2\, F_0 \sin(\Omega t)$ and $T(t)= 50\, l\, F_0 \sin(\Omega t)$. Find the expression of the mass ratio (μ) as a function of (ω_1) and the excitation frequency that renders the pendulum to be stationary under this harmonic excitation. (1.5 points)

(b) Determine the corresponding vibratory motion of the mass M ($x(t)$). (0.5 points)

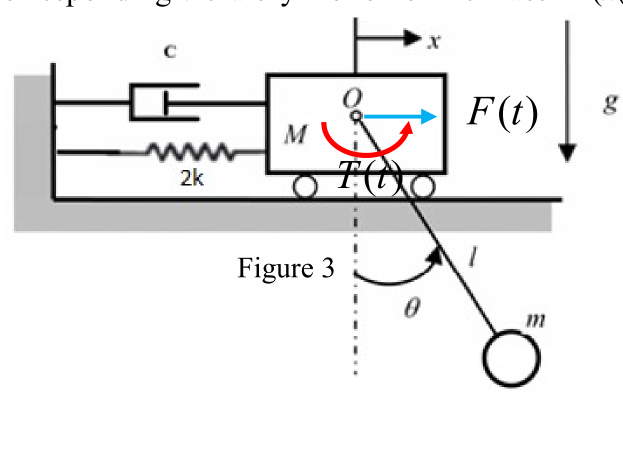

Figure 3

- **Problem 4: (10 points)**

A vibration-isolation block is to be installed in a laboratory so that the vibration from adjacent factory operations will not disturb certain experiments, as presented in Figure 4. If the isolation block weighs 100kg and the surrounding floor and foundation vibrate at 18/πHz with an amplitude of 0.003m.

1- Determine the equations of motion and natural frequency in terms of $\boldsymbol{m, c, k, Y,}$ and ω.

   (2 points)

2- For an undamped system (c=0), determine the value k of the isolation system, in N/m, such that the isolation block will have a particular solution amplitude of only 0.0005m. (2 points)

3- For a damped system, with the value of k=3.468*10^4N/m, determine the damping coefficient c that will allow us to have the same particular solution amplitude (0.0005m). (3 points)

4- Suppose an additional component is rigidly attached to the machine. This component adds an additional mass of 50kg. Determine the particular solution amplitude of oscillation for the undamped system with this additional mass when k=3.7029*10^4N/m. (3 points)

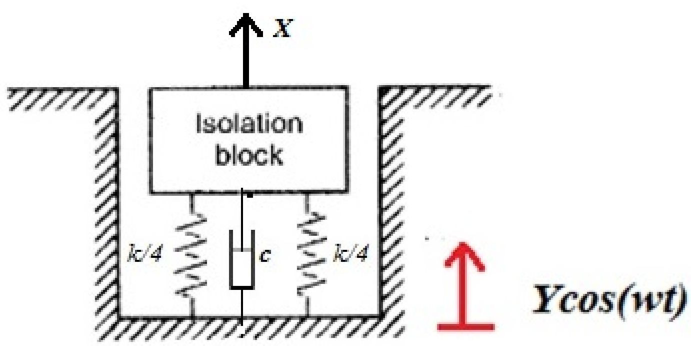

Figure 4