Qualifying Exam, Fall 2020
Fluid Mechanics

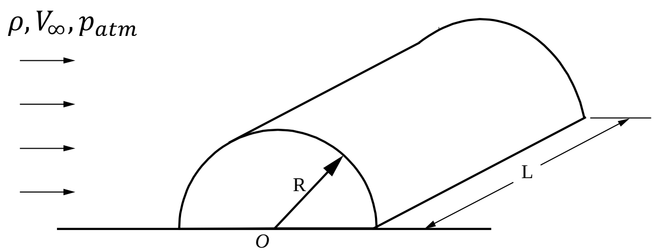
* This is a closed-book test (with a cheat sheet provided), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #____________, #___________, and #____________ to be graded.

1. Wind blows on an airport hangar, which has a semicircular crosssection as shown in the figure. The pressure inside the hangar and the static pressure in the farfield both equal to the atmospheric pressure. Assume 2-D, steady-state, incompressible and inviscid flow, the stream function of the flow can be written as $\psi = V_\infty r\left(1 - \frac{R^2}{r^2}\right)\sin\theta$.

   Find: the expression of total lift force applied on the hangar due to the pressure difference between the inside and the outside of the hangar.

   Equations you may need: $V_r = \frac{\partial\psi}{r\partial\theta}, V_\theta = -\frac{\partial\psi}{\partial r}$



Solution:

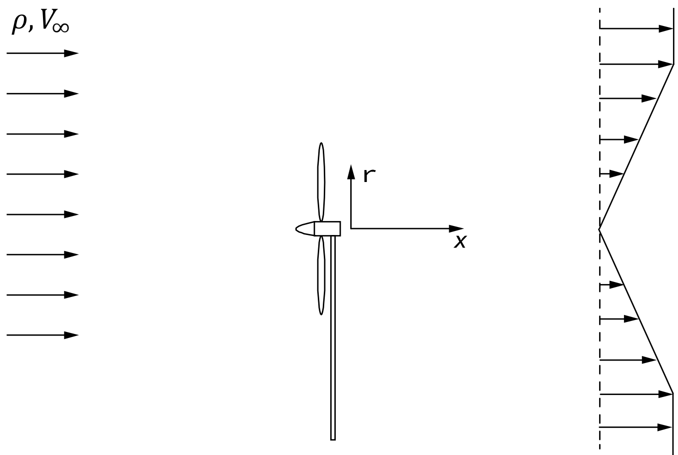
2. A wind turbine (with radius R) in the field is shown in the figure below. Velocity profile downstream the wind turbine is measured as shown. Neglect the ground effect and the influence of the pole.

Find: the expression of the drag force applied on the wind turbine.

Equations you may need:

Conservation of mass: $\frac{\partial}{\partial t}\iiint_{CV} \rho \, d\forall + \oiint_{CS} \rho \, \vec{V} \cdot d\vec{A} = 0$

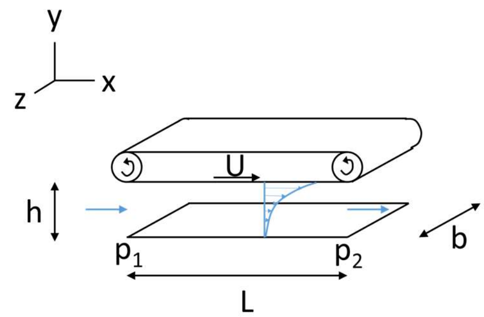
Conservation of linear momentum: $\frac{\partial}{\partial t}\iiint_{CV} \rho\vec{V} \, d\forall + \oiint_{CS} \rho \, \vec{V} \, \vec{V} \cdot d\vec{A} = \sum \vec{F}$



$$u(r) = \begin{cases} V_\infty, & r > 2R \\ \frac{r}{2R} V_\infty, & r \le 2R \end{cases}$$

Solution:

3. Consider a micro-pump for water that is made from a belt that is driven over a flat plate which is at rest:



The length, height, and width of the channel under the belt are L=10cm, h=1mm, and b=10mm.

The belt velocity is U=1m/s and the volume flow rate is Q=1ml/s (=10^{-3} liters/second).

The density and kinematic viscosity of water are ρ=10^3kg/m^3 and ν=$10^{-6}$$m^2$/s.

Assume the flow to be two-dimensional (no variation in z-direction). Neglect the gravitational acceleration, i.e. g=0.

a)

The incompressible Navier-Stokes equations are

$$u_t + uu_x + vu_y = -{}^1\!/_\rho\, p_x + \nu(u_{xx} + u_{yy})$$

$$v_t + uv_x + vv_y = -{}^1\!/_\rho\, p_y + \nu(v_{xx} + v_{yy})$$

where the subscripts indicate partial derivatives.

Assuming that the flow is steady and parallel (the velocity components do not change in x and v=0), simplify the Navier-Stokes equations.

b)

Assume $p_x = \mu\, u_{yy} = const$. Derive an expression for the velocity profile, $u(y)$.

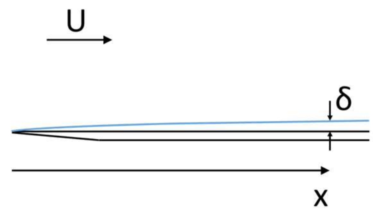
c)

Assume $u = U\frac{y}{h} + \frac{p_x}{2\mu}y(y-h)$. Derive an expression for the volume flow rate per span, Q/b.

d)

Assume $\frac{Q}{b} = U\frac{h}{2}\left(1 - \frac{p_x h^2}{6\mu U}\right)$. What is the difference, $\Delta p = p_2 - p_1$, between the outlet and inlet pressure?

4. Consider a flat plate that is mounted in a wind tunnel:



The wind tunnel freestream velocity and temperature are U=30m/s and T=300K. The density and kinematic viscosity of air are $\rho=1.2 kg/m^3$ and $\nu=1.5\times10^{-5} m^2/s$.

a)

Is the flow compressible or incompressible (M<0.3) ?

At x=1m, is the boundary layer laminar or turbulent ?

b)

Assume the boundary layer profile has the form,

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}},$$

where δ is the boundary layer thickness. Provide expressions for the displacement thickness, $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$, and momentum thickness, $\vartheta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$.

c)

Assume $\frac{\tau_w}{\rho U^2} = 0.0227\left(\frac{U\delta}{\nu}\right)^{-\frac{1}{4}}$ and $\frac{\vartheta}{\delta} = \frac{7}{72}$. Here, τ_w is the wall skin friction. Based on the von Karman momentum equation,

$$\frac{\tau_w}{\rho U^2} = \frac{d\vartheta}{dx},$$

Develop an expression for the boundary layer thickness, $\frac{\delta}{x} = f(Re_x)$. Assume $\delta(x = 0) = 0$.

d)

Assume $\frac{\delta}{x} = 0.373\left(\frac{Ux}{\nu}\right)^{-\frac{1}{5}}$. Compute the boundary layer thickness, δ, at x=1m.