Qualifying Exam 2020

Select three problems

Dynamics

- **Problem 1: (10 points)**

A particle m_1 is constrained to move on a conical surface shown in Figure 1, while a second mass m_2 is constrained to the vertical direction. The two masses are connected by an inextensible string of length L.

1- Choose r, θ as generalized coordinates, define the position vectors for the two masses, and show that the equations of motion using Lagrange's principle can be expressed as (7 points):

$$(m_1 + m_2)\ddot{r} - m_1 r\dot{\theta}^2 \sin^2\alpha - m_1 g \cos\alpha + m_2 g = 0$$

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} = 0, \quad i.e., \ \frac{d}{dt}\left[r^2\dot{\theta}\right] = 0$$

2- Use a constant of the motion to reduce the problem to a single differential equation for $\boldsymbol{r(t)}$. (3 points)

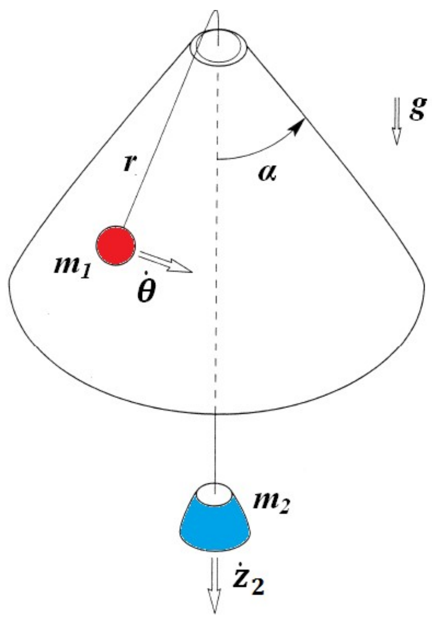

Figure 1

- **Problem 2: (10 points)**

Two particles with mass m are attached by a linear spring with a spring constant k and unstretched length $2r_0$, as shown in Figure 2. Assume that the system dynamics stays in a plane. There is no net external force on the system, so you can use the center of mass of the system is the origin of an inertial frame.

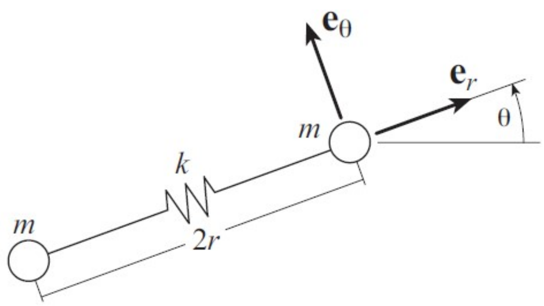

Figure 2

The goal is to obtain the possible motions of the system. Although not obvious at first, this system's dynamics can be reduced to a single ordinary differential equation.

1- Determine the second-order differential equations of motion whose solution would give $r(t)$ and $\theta(t)$ as functions of time; but do not solve them.
   (a) Using Lagrange's principle (2.5 points)
   (b) Using Newton's law (2.5 points)

2- Assume that the initial angular velocity is $\dot{\theta}_0$, the initial radial velocity is $\dot{r}_0$, and the initial separation $2r_0$ is the same as the unstretched length of the spring, determine an expression that relates $\dot{\theta}$ and r and the initial conditions. (2 points)

3- Using the conservation of energy, find a single scalar expression relating $\dot{r}$ and r of the form (3 points)

$$\dot{r}^2 = f\,(r)$$

Vibrations

- **Problem 3: (10 points)**

An absorber of mass M_2 is implemented in order to reduce an engine vibration of an automobile of mass M_1, as shown in the spring-mass representation in Figure 3. If the primary mass is excited by an unbalancing rotating force with a magnitude $md\Omega^2$ and a frequency $\Omega=\theta/t$,

1- Draw the free body diagram (FBD) and apply Newton's second law, prove that the equations of motion of this system can be written as: (2.5 points)

$$\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} \dfrac{2k_1+k_2}{M_1} & \dfrac{-k_2}{M_1} \\ \dfrac{-k_2}{M_2} & \dfrac{k_2}{M_2} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} \dfrac{F_1}{M_1} \\ \dfrac{F_2}{M_2} \end{Bmatrix} \sin(\Omega t)$$

2- What are the expressions of F_1 and F_2? (0.5 points)

3- Consider ($M_1 = m$, $M_2 = 2m$, $k_1 = 0.5k$, $k_2 = 2k$), determine the natural frequencies and associated mode shapes of this system (2.5 points). Then, depict the mode shape of each mode (0.5 points).

4- Consider the general form of the equations of motion (questions 1 and 2),

(a) Determine the proper value of the absorber spring k_2 as a function of M_2 and Ω in such a way the primary system is not vibrating. (3 points).

(b) What will be the vibratory motion of the absorber ($x_2(t)$)? (1 points)

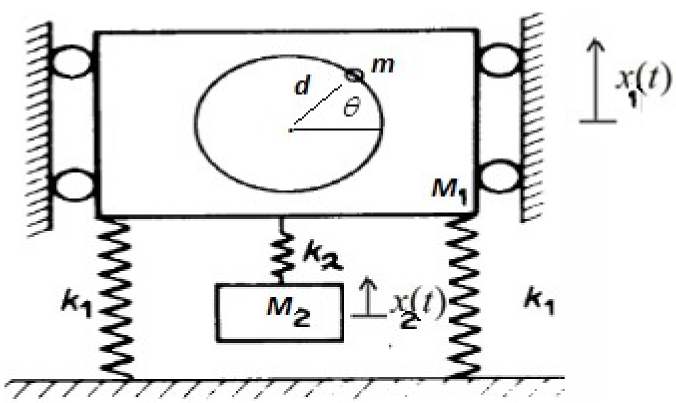

Figure 3

- **Problem 4: (10 points)**

A 2-DOF system that consists of a massless roller grounded by a linear spring and a simple pendulum hinged at the center of the cart is considered, as shown in Figure 4.

1- How many degrees of freedom do this model possess? (1.5 point)

2- Determine the equations of motion of the system using Euler-Lagrange equations_and prove that (3 points):

$$\mathbf{m}\ddot{x}+c\dot{x}+kx+ml\ddot{\theta}\cos(\theta)-ml\dot{\theta}^2\sin\theta=0$$

$$\ddot{\theta}+\frac{\cos(\theta)}{\mathbf{l}}\ddot{x}+\frac{\mathbf{g}}{\mathbf{l}}\sin(\theta)=0$$

3- Linearize these equations of motion when *x* and *θ* around (0,0). Explain. (3 points)

4- Determine the uncoupled natural frequencies. (2.5 points)

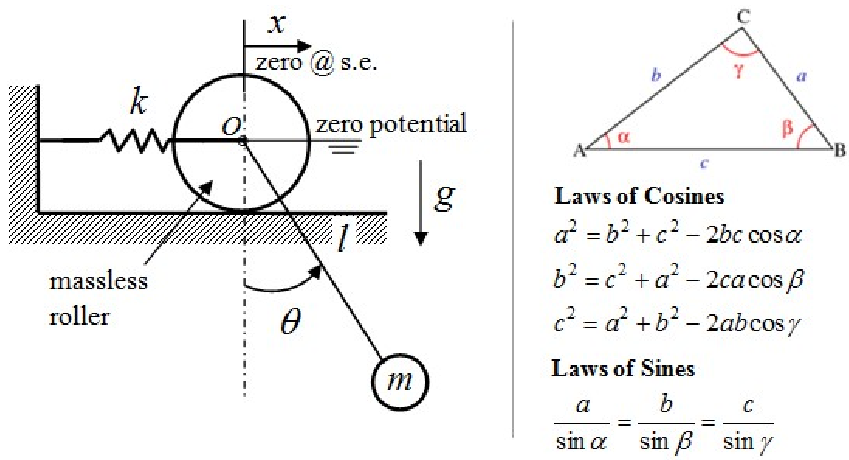

Figure 4