Qualifying Exam, Fall 2020
Control

* This is a closed-book test (with a cheat sheet provided), and no calculator is allowed.
* Work THREE out of the four problems, and clarify which three you want graded.

I want problems #_____, #_____, and #_____ to be graded.

Problem 1. Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 10 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x.$$

(1) Is the system controllable? Why or Why not? (5 pts)

(2) Is the system observable? Why or Why not? (5 pts)

Problem 2. Consider the system

$$\dot{x} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x + u.$$

(1) Compute the system's transfer function. (7 pts)

(2) Is the system Bounded-Input-Bounded-Output (BIBO) stable? Why or Why not? (3 pts)

Problem 3.

Given the system in state equation form, $\dot{x} = Ax + Bu$

where

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(1) Is this system stable? (4 points)

(2) Can you place the system's eigenvalues at $-2, -4, -7$ by state feedback $u = -Kx$, where $K = [k_1 \quad k_2 \quad k_3]$? What is K? or you cannot place? Explain how you concluded. (6 points)

Problem 4. The block diagram of a control system is shown in the figure. Find the region in the K versus α plane for the system to be asymptotically stable. (Use K as the vertical and α as the horizontal axis.) Here, $G_1(s) = \frac{s+\alpha}{s}$ and $G_2(s) = \frac{K(s+2)}{s^2-1}$ (10 points)